DOCUMENT RESUME

ED 168 180 EA 011 458

AUTHOR	Berne, Robert; Stiefel, Leanna
TITLE	The Measurement of Equity in School Finance with an Expenditure Disparity Measure. Papers in Education Finance No. 19.
INSTITUTION	Education Commission of the States, Denver, Colo.
SPONS AGENCY	Office of Education (DHEW), Washington, D.C.
PUB DATE	Nov 78
CONTRACT	OEC-22-78-13030
NOTE	274p.; Not available in paper copy due to marginal legibility of original document
EDRS PRICE	MF01 Plus Postage. PC Not Available from EDRS.
DESCRIPTORS	*Educational Finance; Elementary Secondary Education; *Equal Education; Ethical Values; *Finance Reform; Mathematical Models; *Measurement Techniques; *Tables (Data)
IDENTIFIERS	Federal Expenditure Disparity Measure; Per Pupil Expenditure Disparity Measures

ABSTRACT

This report evaluates alternative approaches to and measures of equity in state school finance plans. It also evaluates the Federal Expenditure Disparity Measure (FEDM). Following an introduction, Part II describes alternative conceptions of equity in education and identifies three components of these conceptions: groups treated, alternative treatments, and criteria for equity. Part III presents numerical models that show how alternative combinations of group, treatment, and criterion can yield different conclusions when applied to a hypothetical school district. In Part IV, the value judgments that underly the alternative conceptions of equity are examined. Since few value judgments are widely held, a number of equity conceptions must be considered valid. Also in Part IV, the value judgments inherent in Per Pupil Expenditure Disparity Measures (PPEDMs) are reviewed and analyzed. In Part V, the theory behind the FEDM and the way the Office of Education applies the FEDM to calculate measures from state data are discussed. In Part VI, recommendations concerning equity measurement in federal regulations and particular procedures used by the Office of Education to measure expenditure disparities are presented. A lengthy appendix considers social welfare theory, social choice theory, equity measures calculations, and the tax price of education. (Author/JM)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

ED168180

U.S. DEPARTMENT OF HEALTH, EDUCATION & WELFARE
NATIONAL INSTITUTE OF EDUCATION

THIS DOCUMENT HAS BEEN REPRODUCED EXACTLY AS RECEIVED FROM THE PERSON OR ORGANIZATION ORIGINATING IT. POINTS OF VIEW OR OPINIONS STATED DO NOT NECESSARILY REPRESENT OFFICIAL NATIONAL INSTITUTE OF EDUCATION POSITION OR POLICY

PAPERS IN EDUCATION FINANCE

PAPER NO. 19

THE MEASUREMENT OF EQUITY IN SCHOOL FINANCE WITH AN EXPENDITURE DISPARITY MEASURE

By Robert Berne
and Leanna Stiefel*

Graduate School of Public Administration
New York University

Prepared for the U.S. Office of Education,
Department of Health, Education, and Welfare
Under Education Commission of the States
Contract No. 22-78-13030

BEST COPY AVAILABLE

EA 011 458

*We wish to acknowledge the consultation of Allan Odden, Director, Education Finance Center, Education Commission of the States.

November 1978

TABLE OF CONTENTS

	Page
I. Introduction	1
II. Alternative Conceptions and Measures of Equity in School Finance	6
A. Introduction	6
B. Groups	9
1. Children: General Arguments	9
2. Children: Legitimate and Illegitimate Differences	11
3. Households	17
4. School Districts	18
5. School Personnel	19
6. Summary	20
C. Treatments	21
1. Introduction	21
2. Criteria for the Evaluation of Treatments	22
3. Alternative Treatments	30
D. Criteria	60
1. Children and Inputs, Outputs or Impacts	60
a. Identical Children	60
i. Introduction	60
ii. Social Welfare Functions and Atkinson's Index of Equity	61
iii. Less Constraining Forms of Social Welfare Functions and Measures of Equity	65
iv. The Level of Measurement Issue	66
v. Value Judgments and Measures of Equity	67
b. Non-Identical Children	72
i. Introduction	72
ii. Univariate Measures	73
iii. Bivariate and Multivariate Measures	76
2. Households and Total Goods and Services	84
a. Introduction	84
b. Atkinson's Equity Index Applied to Households and Total Resources	85
c. Ability-to-Pay Measure	87
d. Ex Ante Households Equity	91
i. Definitions	92
ii. Conceptual Basis for Ex Ante Household Equity	92
iii. Summary of the Relationship Between Tax Prices and House Values Under a DPE	93

	Page
. Hypothetical Numerical Models of School Districts That Illustrate Conflicts in Conceptions of Equity in School Finance	95
A. The Construction of School District Models	95
1. The Model as a Static Snapshot	95
2. The Range of Education and School Finance Variables in School District Models	95
B. Introduction to Model Presentations	103
C. Models for Identical Children's Equity Criteria	105
1. Introduction	105
2. The Models and the Measures	107
3. Summary	109
D. Models for Non-Identical Children Criteria	114
1. Legitimate Differences Among Non-Identical Children and Univariate Measures	114
2. Illegitimate Differences Among Non-Identical Children and Bivariate and Multivariate Measures	119
E. Models for Households Versus Children as a Group	125
1. Introduction	125
2. The Models and the Measures	125
3. Summary	128
F. Models for Household Criteria	131
1. Introduction	131
2. Models and Measures	131
3. Summary	131
7. Conclusions About Equity Measurement Based on Conceptions and Models: What Can and Cannot be Eliminated Based on Theoretical Analyses and Sets of Relatively Acceptable Value Judgements?	136
A. Introduction	136
B. Components That Cannot be Chosen Among on the Basis of Widely Held Value Judgements	140
C. Components That Can be Chosen Among on the Basis of Widely Held Value Judgements	143
V. Value Judgements Inherent in the Regulations That Define and Describe the Federal Expenditure Disparity Measure	147
A. Decisions on the Group	147
1. Pupil Count Alternatives	149
2. Alternatives for Combining Measures for Varying District Organization Types	151
3. Summary	152

	Page
B. Decisions on the Treatment	154
1. Revenues or Expenditures	154
2. Current Dollars	155
3. Accountable Federal Funds and Cost Differentials	156
C. Decisions on the Criterion	160
D. Conclusions About the OE Decisions on Equity Conception	163
I. Recommendations	164
A. Recommended Group, Treatment and Criteria in Disparity Measure	167
1. Group	167
2. Treatment	167
3. Criteria	168
B. Strive Toward Comparability of Treatments and Pupil Counts Across States	170
C. Consider a Graduated Equity Standard as a Replacement for an Arbitrary Cutoff	173
D. Further Refine Choice of Equity Measures Through Concurrent Use of Value Judgment Choices, Empirical Analysis, and Practical Standards of Desirability	176
ppendix 1	
ppendix 2	
ppendix 3	

LIST OF TABLES

		Page
1.	Acceptability of Differences Based on Characteristic	12
2.	Value Judgments for Equity Measures for Identical Children	68
3.	Value Judgments for Equity Measures for Non-Identical Children	81
4.	Plan for Model Presentations	104
5.	Number of Children at Each Per Child Expenditure Level for Distributions A, B, C, D, E, F, G	108
6.	Computation of Equity Measures	110
7.	Ranking of Distributions According to Equity Measures	111
8.	Examples of at Least One Conflict Between Selected Pairs of Children's Equity Measures	112
9.	Distributions A(Non-Price Adjusted) and AA(Price Adjusted)	115
10.	Computation of Equity Measures for Price Adjusted Distribution AA	117
11.	Conflicts When Distribution A is Price Adjusted (Distribution AA)	118
12.	Number of Children, Property Wealth Per Child, and Expenditure Levels Per Child for Distributions N, N1, N2, N3 and P, P1, P2, P3	121
13.	Computation of Bivariate and Multivariate Measures	123
14.	Hypothetical Models for Households and Children	126
15.	Calculations of Equity Measures for Households and Children	129
16.	Hypothetical Household Models, W and Z	132
17.	Calculations of Equity Measures for Households	134

LIST OF FIGURES

		Page
1.	Answers To Value Judgement Questions for Eleven Equity Measures for Identical Children	71
2.	Answers To Value Judgement Questions for Six Equity Measures for Non-Identical Children	82
3.	Widely Agreed Upon Choices in Equity Conceptions	138

I. Introduction

The purpose of this report is to theoretically evaluate alternative approaches to and measures of equity of state school finance plans and to evaluate the Federal Expenditure Disparity Measure (FEDM) as a particular equity measure.

After this introduction, the broad concept of educational equity is specified and discussed so that the FEDM can be analyzed within a general framework. In the theoretical discussion of equity, three components of equity are identified which are then used to define equity conceptions. Once alternative equity conceptions are identified, hypothetical numerical models of sets of school districts are constructed in order to demonstrate, in a simplified manner, that the alternative equity conceptions can lead to conflictual (or contradictory) rankings. These examples are set up to show that the different value judgments embodied in the alternative equity conceptions can, potentially, make a difference.

Next, the conclusions about equity measurement that can be drawn from the theoretical discussion and hypothetical models are summarized. Again it is demonstrated that value judgments play a key role in determining which equity conceptions are preferred. In addition, the particular value judgments inherrent in Per Pupil Expenditure Disparity Measures (PPEDMs) are highlighted. The FEDM a PPEDM and the specific value judgments embodied in the FEDM are considered next. In order to utilize the FEDM to measure disparities in the States the Office of Education has published

"instructions" and the value judgments inherent in the FEDM, as specified in these instructions, are discussed.

Finally, this report concludes with a set of recommendations on equity measurement in general and the FEDM in particular. In addition, measurement issues and the use of a graduated scale in place of an absolute equity cutoff are explored. Three appendices are also included in this report which discuss the application of social welfare and social choice theory to the measurement of educational equity, detail the mathematical formuli used to calculate the equity measures, and analyze the tax price of education in relation to educational equity.

The remainder of this report is divided into five parts. Part II describes alternative conceptions of equity in education in considerable detail. A conception of equity is composed of first, the group from whose perspective equity is evaluated, second, the treatment that is to be equitably distributed and, third, the criterion by which equity can be measured. Part II begins with a discussion of the groups that serve as a focus for an assessment of educational equity. Children and households are the prominent groups considered. Next, alternative treatments including school inputs, outputs, and impacts are evaluated as treatments in an equity conception. Finally, the third component in the equity conception, the criterion, is discussed. The nature of the criteria are such that they are presented with particular combinations of groups and treatments. The explanation of the criteria includes an analysis of the value judgments that are embodied in each criterion.

The various combinations of group-treatment-criterion that define equity conceptions described in Part II can lead to conflicting equity evaluations when applied to sets of school districts. These conflicting equity evaluations are illustrated in Part III where a number of hypothetical numerical examples are constructed. The models are constructed as simply as possible to show how the selection of alternative combinations of group, treatment, and criterion can yield conflicting conclusions when used to rank a set of hypothetical school districts in terms of equity.

Parts II and III enumerate a large number of equity conceptions and show that they may yield different conclusions. Furthermore, the different conclusions stem, in part, from the value judgments built into the various equity conceptions. In Part IV these value judgments are more closely examined to determine whether certain equity conceptions can be seen to be more preferable than others. It turns out that only some value judgments can be considered "widely held" so that a number of equity conceptions, rather than a single conception, must be considered unless other information is used to more narrowly restrict the value judgments. Also in Part IV the Per Pupil Expenditure Disparity Measures (PREDM's) are singled out for further discussion in terms of "widely held" value judgments. Since the FEDM is a PPEDM, the value judgments embodied in the PPEDM's are also embodied in the FEDM.

The FEDM has certain particular characteristics that distinguishes it from other PPEDM's and these specific characteristics are reviewed and analyzed in Part V. The conceptual nature of the

FEDM is considered and the manner in which the Office of Education applies the FEDM to actually calculate measures from state data is discussed. Items treated in this part include the pupil count utilized in the FEDM, the way in which data from multiple district types are combined, the measurement of revenues and expenditures, and the criterion in the FEDM.

Part VI presents the recommendations which are drawn from the analyses accumulated in Parts II through V. First, recommendations are made concerning the general issue of equity measurement in Federal regulations. Second, a series of recommendations are made regarding the particular procedures that the Office of Education plans to use to measure expenditure disparities. The recommendations are directed at the treatments and pupil counts, the way the disparity measure can be used to judge equity, and the particular criterion utilized in the FEDM.

The three appendices attached to this report are included to provide additional depth in certain areas that can be examined separately from the primary analysis in the report. In economics, social welfare theory and social choice theory have developed theories and techniques that can be utilized to measure the equity in society in general and in income distributions in particular. In Appendix 1 social welfare theory and social choice theory are reviewed so that the application of these areas to education equity can be seen. As it turns out, certain key dilemmas in education equity are also unresolved in the social welfare and social choice fields. Appendix 2 contains the mathematical formuli that are needed to calculate specific equity measures. These are presented

in an appendix so that they can be referred to in one place. Finally, since the purpose of this report is to analyze the expenditure disparity measure, only limited attention has been paid to the price of education faced by a household in a school district. In Appendix 3 the issue of the tax price of education is considered in more depth and related to various concepts of education equity discussed in this report.

II. Alternative Conceptions and Measures of Equity in School Finance

A. Introduction

In this part of the paper we set out several alternative conceptions of equity in school finance. The alternatives often depend on value judgments, which we strive to make clear. The sections in this part are conceptual; illustrative numerical models of each conception are presented in Part III and specific applications to Per Pupil Expenditure Disparity Measures (PPEDM) and the Federal Expenditure Disparity Measure (FEDM) follow in the last parts of the paper.

Conceptions of equity, rather applied to education or to any other distribution of society's goods or ills, require the specification of three components. Those three components are the group or unit of analysis, the treatment, and the equity criteria. The three components answer the questions of who, what, and how the equity conceptions is about. Sections of this part of the paper are devoted to each component as the following outline of the major sections shows.

Outline of Part II

A. Introduction
B. Groups
   1. Children: General Arguments
   2. Children: Legitimate and Illegitimate Differences
   3. Households

4. School Districts
5. School Personnel
6. Summary

C. Treatments
   1. Introduction
   2. Criteria for the Evaluation of Treatments
   3. Alternative Treatments
      a. Schooling inputs
      b. Schooling outputs
      c. Schooling impacts

D. Criteria
   1. Children and Inputs, Outputs, or Impacts
      a. Identical Children
      b. Non-Identical Children
   2. Households and Total Goods and Services
      a. Introduction
      b. Atkinson's Index
      c. Ability-to-Pay Measure
      d. Ex ante Household Equity

Before beginning the descriptions of alternative components an equity conceptions, we take a bit of space to further develop the idea of each of the three components.

The group specifies the individuals to be included in the distributional concerns. Very rarely does any discussion of equity include all the individuals, both present and future, of the world. Often the currently living people of a country are focused upon and sometimes the group is further narrowed by the specification of a regional, age, sex, race or other identification. We will discuss the value judgments inherent in various choices of a group for educational equity.

The identification of a treatment indicates the thing of value that is distributed more or less equitably among members of the relevant group. The choice of the treatment is important since

quity in the distribution of something unwanted or of no value is not very meaningful. Implicit differences in the treatment of concern often lead to confusion in discussions of equity because the choice of the treatment can change one's view of the equity of a situation.

Finally, the application of the treatment to members of the group requires the specification of an equity criterion. The criterion is one of the most important components of a conception because many unresolved philosophical problems are embedded in the choice of a criterion. The section on the criteria identifies the value judgments inherent in different criteria and the statistical measures representative of each set of value judgments.

Throughout this paper we speak of equity conceptions, ather than equality conceptions. Equity is a broader concept than equality. Equity includes notions of both equal treatment of equals and unequal treatment of unequals, while equality is concerned only with equal treatment of equals. Equality will not always be equitable, especially if group members are not the same and should be treated differently. This difference will become clearer as we work our way through the criteria sections for identical and non-identical children.

B. Groups

There are four groups that could serve as the focus for a conception of equity in education. The four possible groups are the children who "receive" the education, the adults or households who possibly benefit from and pay for the education, the school districts which are the political unit through which many education decisions are made, and the teachers and other employees who provide the education.

1. Children: General Arguments

The children alone are an appealing choice for a number of reasons. First, because they are society's link with the future, there is a strong case for providing them with adequate and equitable beginnings as a way to influence positively the structure of society's future. Education and children interact in this argument. Education is a uniquely important service because of its perceived effect throughout life and children are important because most people believe that their life chances can to some extent be influenced by people and events outside their immediate families. Second the benefits of education flow most immediately and directly to the children. They are the ones who spend twelve or more years of their lives in the classrooms and thus are exposed to the immediate effects of the educational experience. In addition, children embody whatever future effects the educational experience may bring and although other groups may derive benefits from those embodied in children, the children

themselves gain directly. Finally because children are in many ways unable to fend for themselves and, in particular, unable to make good (if any) choices in their self-interest, society may have an obligation to look out for them and to provide equitable treatment.

The arguments for concentrating on children as the unit of analysis, (children as an intergenerational tie, as direct and immediate beneficiaries, and as incapable of self-interested decisions) are general ones that would lead to the inclusion of both public school and private school children.[/] Yet because United States governments currently have so much more financial and regulatory influence over public schools, there is a temptation to concentrate on public school children alone. A broad view of equity would be hard-pressed to accept this. Government regulations and financial arrangements are variables affecting equity and can hardly be considered parameters in any general conception. As a practical matter one might empirically find that private school children are always treated at least as well as public school ones and that a relatively small percent of the children are in private schools. In this case leaving private school children out of the analysis will neither hurt them nor have much impact on public school children's equity. The problem is that some private schools may not be as "good" as public ones or if private schools are as good as public ones, over time more children may shift from public to private schools.[/*]

Children: Legitimate and Illegitimate Differences

Children as a group are not homogeneous, but rather they vary on a number of dimensions. The differences among them are important when the second and third parts of an equity conception, the treatment and the criterion, are combined with the group. If differences among children are considered "legitimate," then the equity criterion may specify a different distribution of treatments than would be specified if children are either identical or if their differences are considered "illegitimate". In this section several categories of differences among children are identified. Each category is subsequently discussed and individual characteristics within the categories are classified as legitimate, controversial, or illegitimate. It should be noted that the classification of children's differences into legitimate and illegitimate is, in the end, a value judgment. For many differences, however, either because of court decisions or frequent legislative recognition, it is fairly clear in which categories most people currently place the difference. These clear-cut differences are the ones classified in this paper. The less clear-cut ones are identified as controversial.

Differences among children can be categorized as those due to characteristics of the individual children, those due to characteristics of the districts where the children reside, or those due to school programs in which the children are enrolled. [1] Table 1 summarizes the classification of characteristics within the three categories by their degree of legitimacy. The remainder of this section expands on these classifications.

Table 1

Acceptability of Differences Based on Characteristic*

Type of Characteristic	Legitimate	Controversial	Illegitimate
Student based	Learning disabilities Inadequate preschool preparation Health problems Severe physical or mental handicaps		Sex Race Ethnicity
District based	Technological (costs) Economies of Scale Safety Production Transportation Exogenous influences on price	Municipal overburden	Property wealth Endogenous influences on price
Program based	Student mandated Handicapped	Student chosen Vocational education College prep.	

*Classifications of all characteristics contain some value judgments. The assignments in the table reflect the authors' judgments of currently widespread agreements.

a. Children's Characteristics

Children's characteristics frequently given special consideration in legislative finance programs include special learning disabilities, such as those that result when English is a second language; poor pre-school preparation that sometimes results from an impoverished upbringing; and health related problems such as physical and severe mental handicaps as well as those related to drug abuse. Most finance programs give recognition to the necessity for higher educational resources in order to meet minimum output goals for these kinds of children. On the other hand, children's characteristics such as race, sex and ethnicity are generally not considered legitimate differentiating characteristics in terms of educational treatments. The illegitimacy of these differences probably results from broad adherence to the principles of the Fourteenth Amendment to the Constitution as well as from the plethora of state court cases confirming race and sex as "suspect classifications." The age of the child and his/her grade level do not clearly fall into either category. Many people think that either younger or older children or children in different grade levels need different treatments, but many others think it unnecessary or even discriminatory to classify by age and there is not universal agreement on whether the higher or lower grades should be weighted more heavily.

b. District Characteristics

Legitimate differences based on district characteristics generally result from a need to provide more resources in order to achieve a constant level of output because of technological

factors or a need to provide more dollars to achieve equivalent resources because of price differences. Technological factors include economies and diseconomies of scale due to the number of pupils in the district and different resource requirements for the production of safety and of transportation, etc.

Probably, the most often discussed factor in the category of technological characteristics is district size measured by number of children. The question can be posed as follows: does an equivalent amount of expenditures or resources per child in districts of varying size produce different outputs? It may be that smaller districts have smaller classes since they have fewer children per grade to allocate to classes but it would only be inferred that costs are higher for the small district if there were no commensurate benefits derived from smaller classes. Outside of the classroom there may be higher non-instructional costs for smaller districts due to certain economies of scale and for larger districts due to higher coordination costs, however, there is still the question of whether the services to the children vary as well.

If cost differences among districts of varying size reflect output quantity or quality differences then size adjustments are not appropriate in equity measures; however, if cost differences among districts of varying size do not reflect these differences then size adjustments are appropriate in equity measures. In reality the "truth" probably lies somewhere in between these two extreme positions and existing research cannot give the precise adjustments. The question of an adjustment for size becomes to some degree a value judgment.

Price differentials are conceptually different from technological or cost differentials. Technological or cost differences result from the need for more real resource usage to achieve a given quality and quantity of output. Price differentials are the result of differences in the price per unit of equivalent amounts of resources.[/] When price differentials are considered legitimate, there is usually a distinction between the causes of the differentials. Causes outside the district's control, such as geographic location or district student composition are considered legitimate while causes within the control of the district such as laxity in collective bargaining are illegitimate as discussed in more depth in future sections. The construction of price indexes that can separate the causes is in the development stages. Conceptually, there would seem to be a strong case for including uncontrollable price differences as legitimate differences, but in practice because price indexes cannot yet be reliably estimated, few states take account of such differences.

Sometimes the "urbanness" of a district is considered a legitimate differentiating characteristic for children. Often the arguments that urban districts require more resources per unit of output revert back to already discussed characteristics of children (higher proportions of handicapped, bilingual, poorly prepared, etc.) or of the district (diseconomies of scale, more resources needed per unit of security, etc.), in which case urbanness is merely a proxy for these other differences. Some people think that in addition to the children and district characteristics, urban districts require more resources because of "municipal overburden".

Municipal overburden is a measure of the "needs" of a district to finance services other than education. There is no clear-cut concensus about the inclusion of this measure in the legitimate category, in part because it represents a problem with revenues rather than expenditures and in part because the problem may affect services other than education more than it affects education.

Property wealth of a district has received abundant attention inside and outside of the courts in the 1970's. Most people now classify district wealth as an illegitimate, "suspect", characteristic. Efforts to achieve fiscal neutrality are aimed at eliminating the relationship between spending per child and property wealth in a district. Property wealth is clearly considered an illegitimate characteristic upon which to differentiate the treatment of children.

c. School Program Characteristics

A final category of differentiating characteristics is based on the kind of program the child is enrolled in. For example, differences in resources for vocational education versus college preparatory curriculums are often considered legitimate. The differences are usually justified on the basis of higher costs for vocational programs. The acceptance of this kind of difference on the basis of cost differentials is on fairly shaky grounds conceptually because costs are always a function of the quality and quantity of output produced. Vocational educational programs need not be costlier per child if output levels are set lower. For example, class size or time spent on machines is a variable

affecting cost and output. If predetermined output levels are established and anything else is considered inequitable, then costs would be a legitimate difference. This use of program differentials as legitimate also implies that children's choices as to program are acceptable reasons for varying resources to achieve equity. The previously considered differentials were outside the immediate control of the child (i.e., handicap) or district (i.e., size) and thus program enrollment differentials have a different conceptual basis.

The just-discussed classifications of differentiating characteristics are all summarized in Table 1. It should be reiterated that all the classifications are to some extent value judgments and the particular choices made in the paper reflect the authors' judgments about currently acceptable classifications.

3. Households

All individuals are a second plausible choice for the group or unit of analysis. One might argue that adult as well as child well-being is important to society, or that everyone counts. Then, given a scarcity of resources and a multitute of desirable consumption and investment opportunities, of which children's education is just one, the distribution of all goods and services among all members of society might be considered an appropriate unit of analysis.

Another version of this argument might view children as the property of adults, but of no importance in their own right. The property rights could extend to childless adults. Because parents most often live together with their children, all adults and children could be grouped into households and the households

could serve as an appropriate unit of analysis. Then children's education would enter into household utility functions along with all other public and private goods and services and the focus would then be on the distribution of total consumption (or the welfare derived from it) by households. The unit of analysis would logically include households without children as long as the distribution of goods and services other than education were of concern and/or childless households received benefits or burdens from the provision of children's education.

Looking ahead, it is clear that the logic of the use of households as a unit of analysis, argues favor of a treatment that measures household total welfare. To use households as a unit of analysis, and children's education services alone as a treatment, is inconsistent with the rationale for households that is based on the importance of their overall well-being, unless children's education is the only desirable good that can be produced.

4. School Districts

The school district is sometimes used as the unit of analysis in discussions of educational equity. There are two arguments that can be made for this choice. One is that political jurisdictions as presently formulated combine groups of people (children or households) in ways that make the groups comparable. Behind this argument might be the idea of a political welfare function that assigns weights to individuals and then combines the individuals into groups whose aggregated (summed) weights are identical. In other words, larger districts would implicitly have lower valued individuals.

A second argument would be that the leaders of the districts, either on the provision of funds side (legislators) or on the use of funds side (school boards, superintendents) are the real unit of analysis and the school district is only a convenient representation of these leaders.

Either argument is weak in terms of an educational equity conception, but one suspects that the distribution of spoils among legislators is the primary motivation for the prevalent use of the district as the group. This latter distribution may be important, but not necessarily for education equity, so it is rejected.

5. School Personnel

A final possible unit of analysis is the school district personnel, primarily teachers. Although generally it is the recipients of a service who are of concern, it might be possible to make a case for using the providers of the service. This would be especially true if those providers had no options (inelastic supply) and/or there were some other reason to especially identify them. Clearly school personnel, given their levels of education and expertise, have options. Other reasons for considering them special would seem to derive from the value of children or households as the unit of analysis. For example, it might be desirable to treat teachers fairly in order to minimize the incentives for good teachers to migrate to "desirable" schools. But the reasoning relates to the desire to have a fair distribution of "good" teachers among children and households, not to treat teachers per se fairly.

6. Summary

The viable options for the choice of a group are: public school children, all children, households with children, and all households. All children and all households are more encompassing than public school children or households with children. The choice between the more encompassing and the less encompassing groups depends on value judgments about the importance of those who are ignored when the group is narrowed. The choice between children and households is also a value judgment that depends on one's view of education as either a uniquely important service that benefits primarily children or as a service that benefits all members of society, rather directly or indirectly. Because value judgments are involved in the choices among the four viable groups, the superiority of one cannot be objectively determined.

C. Treatments

1. Introduction

The first component of a conception of equity, the groups upon which the conception of equity could focus, was described in the previous section. In this section the second component of the equity conception, the treatments that can be applied to the groups, is discussed. In other words, first equity for whom is examined, now equity of what. As was the case earlier, the purpose of this section is to present alternative treatments and assess these according to a common framework so that the advantages and disadvantages of each treatment are articulated and clarified.

Although we will present criteria to use in the assessment of the alternative treatments, the individual criteria and the evaluation of the treatments according to these criteria are often the expression of particular value judgments. Our aim is not to impose a set of values but instead to point out those that are embodied in each treatment. Thus at times we often stop short of labelling a characteristic of a treatment as an advantage or disadvantage since one person's advantage may turn out to be another person's disadvantage.[/]

The alternative treatments considered in this section are not restricted to the particular treatment (or treatments) specified in the Federal regulations. Nor are the treatments limited to alternatives specified in "school finance plans". Instead,

we are starting with a broader definition of the education system which includes aspects of the school finance plans as well as the specific definition employed in the regulations. Taking this broader view will enable us to more effectively evaluate the Federal government's approach which is our eventual goal.

This section proceeds first by specifying a set of criteria by which we can evaluate each of the treatments and second by considering each of the treatments measured against the set of criteria. Although certain treatments will be more naturally linked with a sub-set of the groups discussed in the previous section we will examine the treatments separately, as was the case for the groups, and then link particular groups and treatments the following section where specific equity criteria are formulated. We turn now to the set of criteria used to assess the alternative treatments.

2. Criteria for the Evaluation of Treatments

We have identified five criteria that can be used to evaluate alternative treatments that will form a component of a conception of equity. The first three criteria are relevant for treatments in an equity conception in any circumstances. First, we examine whether the treatment is what we desire to be equal at a conceptual level. This criterion forces a review of the values that are inherent in a particular treatment. Second, we evaluate whether there are methods that can be used to measure the treatment in question. Some conceptually preferred measures may defy reasonable measurement. The third criterion asks whether the treatment in question is linked conceptually and empirically to what we desire to be equitable assuming the treatment itself is not exactly what we desire to be equitable.

While these three criteria can be applied to any treatment that is a potential component of an equity conception, we introduce two additional criteria that are relevant if we are evaluating the treatment for inclusion in an equity conception that will become part of Federal regulations to allocate resources among the states. First, if one purpose of the Federal program is to provide an incentive for states to move in a direction valued by the Federal government, do the states have the leverage to influence the treatment? Note that this criterion would not be necessary if the purpose of the legislation were to compensate the states for certain (adverse) conditions that prevailed in the state at a particular point in time. In this latter case it would be appropriate to use a treatment that was for the most part beyond their control such as adverse weather conditions or unemployment. The second criterion that should be considered if the equity conception is to become part of a Federal program is whether there are comparable data available for measurement of the treatment for education in all states. While this criterion is rather pragmatic, as opposed to conceptual, it is an obvious consideration for pending or on-going legislation. The five criteria are more fully explained in this part before we examine the specific treatments.

a. Is the treatment what we desire to be equitable?

This first criterion, or question, forces an examination of the meaning of the treatment in light of our notions of equity. A priori, we do not expect agreement or unanimity in the preferences for one treatment over another, but leaving aside the other

criteria, there may be different choices at a conceptual level.

One way to sort out alternative conceptual preferences is to consider three types of educational or schooling treatments specified as schooling inputs, schooling outputs, and schooling impacts.[1] Schooling inputs can be thought of as those resources that are combined in a variety of ways to educate children in schools. As we will see in the next section there are various ways to measure these inputs but at a conceptual level, a preference for equity of school inputs is consistent with the idea that all children should have access to, or the opportunity of access to, a similar set of resources. Schooling is publicly provided and thought to be a key determinant of outcomes in later life so that inequitable schooling inputs could lead to a potentially unjust advantage for some children. Furthermore, schooling inputs may be directly related to the process of schooling and it may be unjust for children to spend twelve years of their lives in differentially satisfying publicly provided institutions.[1*]

A conceptual preference for inputs as the treatment in a conception of equity is not necessarily synonomous with equal schooling inputs. Equitable schooling inputs may be consistent with inequality of inputs if, for example, certain student characteristics such as educational or socioeconomic background, or native language or certain school characteristics such as size or program type are judged to be legitimate characteristics for differential treatment. Also the equity of schooling inputs may refer to an equitable <u>process</u> that may or may not lead to an equal outcome.

The recognition of different characteristics for judging the equity of inputs stems from a concern for schooling outputs. Where equal schooling inputs are believed to produce different outputs, differences in inputs may be judged to be equitable. As an alternative to focusing on differences in inputs, attention could be directed at the category of schooling outputs. Schooling outputs that could be considered in this conceptual category include achievement or skill levels, graduation rates, or college attendance behavior. If we leave measurement and data problems aside and treat them in sections to follow, a conceptual preference for equity of schooling outputs differs from a preference for inputs since the use of outputs does not require specific attention to the factors that cause output differences. That is, by focusing on inputs we must explicitly pay attention to factors that may lead to inequitable outputs whereas the use of outputs automatically includes a wide range of factors that directly influence the treatment. For example, if we believe that bilingual students should receive additional inputs, then we must explicitly take this factor into account when we employ an input treatment in an equity conception. However, if we use an output treatment in the equity conception then a factor such as bilingualism may not have to be explicitly identified in order to measure equity.

Note that a conceptual preference for schooling outputs as the treatment for equity concerns does not necessarily imply equal

outputs. In addition to equal schooling outputs, equal absolute increases in output levels or equal marginal increases in output could enter into our conception of equity.[/]

A conceptual preference for schooling impacts, the third and final type of treatment we consider, is consistent with a more societal, as opposed to school, focus. Schooling impacts could include the current utility received from allocating resources to schooling and all other goods, or the future impacts that schools are believed to affect. These future impacts could include, for example, earnings or income, social status, or satisfaction or utility. This conceptual treatment is more removed from the school itself and takes into account more student, school, and societal factors within the treatment itself. Furthermore, there is the assumption that equity of these variables is desired and that there is some relationship with schools. Although this conceptual treatment may be viewed by many as a "straw-man", its inclusion forces us to explicitly question whether or not we are concerned with the impact of schooling, the allocation of resources to schools, or the outputs of the schooling process.

b. Are methods available to measure the treatment?

The desire to examine and not merely discuss the actual equity of education or "school finance plans" leads us to a consideration of measurement. Measurement includes a reliable and valid quantitative assessment of a treatment, preferably in a single unit of measure that has cardinal (or interval measurement properties), and that is comparable across situations. In

almost all situations we strive for a measurement methodology or technology that is reliable and valid. The concept of reliability is closely related to the notion of measurement error.[/] A somewhat different way of viewing reliability is in terms of stability, accuracy, or precision.[/*] On the other hand, the concept of validity is concerned with the question of whether the measure is capturing what we are intending to capture.[/**] Note that a measure can be highly reliable without being valid. For example, temperature can be reliably calculated but it may not be a valid measure of perceived chill unless wind conditions and humidity are taken into account.

The need for a single measure for the treatment comes about since we will be involved in comparisons of one unit of analysis with another and it is desirable to avoid conflicts in the comparisons that might result if two or more measures of a treatment were used. Therefore, if the treatment has more than one component, we will need an index that can include contributions from the various components. However, the existence of a single measure of a treatment is a necessary but not a sufficient condition for comparability across units. In addition, we need to be sure that the measures themselves are comparable across units. This becomes particularly important if we are trying to measure satisfaction or utility. In this case the problem of interpersonal comparisons must be confronted.[/***]

Next, the measure should have cardinal properties (interval scale properties)[/****] so that differences between two measures are meaningful.[/*****] This property is considered more demanding

than a simple ranking. While the property of cardinality is not required for all equity conceptions, it is necessary for most so that we include it in the measurement criterion. Thus we have a desire for a valid and reliable single cardinal measure that is comparable across units of analysis.

Finally, after the above-mentioned measurement criteria are considered, there still may be some additional value judgments that come into play. For example, when we measure "students" we have a number of options such as membership versus attendance based figures. These alternatives may turn out to be similar in terms of reliability, validity, cardinality, comparability, etc., yet legitimate differences over which student measure to utilize may still exist.

c. Is the treatment linked to what we desire to be equitable?

This third criterion becomes important in cases where it is not possible or appropriate to use the conceptually preferred treatment in a conception of equity. This situation may occur, for example, when measures of the preferred treatment are not sound, the data for the preferred measure are unavailable, or the incentives that follow from the use of the treatment are not predictable or likely to be beneficial. If the treatment used in the equity conception is not the most preferred, then we can examine the links from the utilized treatment to the preferred treatment both conceptually and empirically. Conceptual linkages are based on our theory of how the education system and society operate. However, just because we believe a linkage exists does not mean we can empirically demonstrate its presence. Therefore,

we are obligated to review the empirical literature to determine whether the linkage is empirically robust, unverifiable or somewhere in between. Our evaluation on this criterion will be selective and we will often survey reviews by others; in some cases we will only show that there is reason to believe that the linkage may exist under certain circumstances.

d. If the Federal legislation is intended to motivate certain behavior, is the treatment under the control of the states and local education agencies?

If one purpose of the Federal legislation that includes standards of equity is to motivate states to formulate policies and enact programs that move their education systems towards greater equity, then the treatment used in the legislation should be capable of being influenced by state policies. Common sense, as well as theories of motivation at the individual level,[/] suggest that the target of the motivating instrument specified in the Federal regulations should be in a position to do something about the treatment in question <u>if</u> the motivating instrument is to have an impact. Otherwise, the instrument will be rewarding the target for the condition of a treatment that is not controllable and hence will not be likely to change the target's behavior. This does not imply that <u>all</u> Federal programs should be based upon controllable treatments. This also does not imply that the intent of section S d(2) of P.L. 81-874 as amended by P.L. 93-380 is to <u>motivate</u> behavior. However, <u>if</u> the desire is to motivate behavior, then the remarks in this section are applicable.

c. Are there high quality data available at a national level to measure equity using the treatment in question?

Since the Federal regulations are to be utilized immediately, there must be data available to calculate the existing level of equity in each state. If we assume that the treatment can be measured, we also have to insure that the measurement has been carried out, or we should ascertain the political and economic feasibility of obtaining the necessary data. The purpose of the assessment of equity is to measure the degree of equity throughout the state so that data from all school districts, students, households, etc. is desirable. If this is not possible, then the particular sampling procedures employed should be carefully scrutinized. In the long run, if we are not concerned with the cost, data can be made available for any treatment that can be measured. But resources are scarce and the Federal Government is interested in using equity standards prior to the long run, so data availability is a valid criterion.

These five criteria are used in the next section to establish a framework for the evaluation of a number of possible treatments. In addition, there are several issues that come up in the context of a specific treatment and these will be discussed as well. Finally, we do not anticipate that the criteria will lead to one best treatment. Instead, there are likely to be tradeoffs among the criteria and the selection of a best treatment will rest on the importance attached to the set of criteria.

3. Alternative Treatments

The first criterion suggests that there are three broad groupings of treatments that can represent alternative conceptual

preferences for what we desire to be equitable. A case could be made for the equitable distribution of schooling inputs, schooling outputs, or schooling impacts. Since, at a conceptual level, the selection among the three is a value judgment we organize the discussion of alternative treatments around these three groupings. We discuss treatments from each of the three possibilities and evaluate them on the other four criteria. Furthermore, the discussion in this section focuses on possible treatments in a conception of equity and while measurement issues are raised, particular measurement techniques are deferred until later in the report.

a. Schooling inputs

To date, the majority of analyses of education equity utilize some measure of schooling inputs as the treatment. Conceptually, schooling inputs could include the tangible and intangible resources that are combined and used to produce education in schools. One possible way to measure schooling inputs is to focus on the resources directly and a second approach is to focus on the dollars that purchase the resources. Dollars are not necessarily the same as resources across school districts since districts may face different prices for identical resources. Furthermore, resources may produce different amounts of outputs across districts due to different cost factors or efficiencies in use of resources. Methods have been developed to modify dollar measures to take price differences into account and these are discussed below in addition to pure dollar and resource measures.

Almost all schooling input measures are expressed as a ratio, usually in the form of the total input measure for the district

divided by a student measure for the district. Before we consider individual treatments, several measurement issues can be highlighted. Although we delay our discussion of particular student measures until later in the paper, there are three measurement issues that are important. First, there are a number of ways to "count" students including enrollment, membership, and attendance based figures. An issue to consider is whether all states should (or could) use the same measure and whether one measure is conceptually preferred. Second, there is the issue of student weights. Various weighting systems have been devised and some have been incorporated in school finance plans. The basic measurement question again is whether a weighted or unweighted student measure should be used and whether there should be uniformity across states or in states over time. The third measurement issue is concerned with the relationship between the numerator and the denominator in the input measure. The issue, stated as a question, is whether the resources as measured correspond to the students as measured. For example, if summer school resources are included in the numerator are summer school students also included in the denominator? There are a large number of questions such as this that can be asked, but we defer discussion of these issues until later in the paper. In this part we assume that these three measurement issues have been settled.

In the remainder of this part we discuss dollar measures including revenues and expenditures, price adjusted dollar measures, physical resource input measures, and quality adjusted resource measures.

Dollar inputs measured as revenues or expenditures

Two measures of dollar inputs to the education system are revenues, the dollars that are received by the educational unit such as the school district, and expenditures, the dollar value of resources that are purchased by the educational unit. In this discussion of dollar measures we assume that a particular measure of either revenues or expenditures has been agreed upon. But before we assess the dollar measure according to our criteria, it should be pointed out that there are real differences between revenues and expenditures particularly when we measure something other than total revenues or expenditures. Revenues can usually be examined by source including local, state and federal sources. These individual sources can be divided further by type of program such as general and categorical for state sources or Title I and impact aid for Federal sources. Expenditures can be identified by purpose such as operating debt service and capital expenditures. The operating expenditures category can be further divided into instruction, utilities and maintenance, transportation, food service, etc. and instructional expenditures can be sub-divided again.[1]

Thus the selection of the precise revenue or expenditure measure is not a trivial question even after deciding to use a dollar measure, but the details of this issue will be discussed more completely later in the paper. In this discussion we focus on the use of a dollar measure without specifying which one.

Dollars obviously can be measured but certain measurement issues do arise. Dollar measures can be recorded reliably due to the development of accounting and reporting systems. Most reliability type problems can be attributed to definitional

difficulties but these should be minor within states and controllable across states.

If inputs measured in terms of resources are what we desire then dollar measures may not be perfectly valid. Dollars do not measure resources across educational units if these units face different prices for identical resources. These price differences may be more pronounced in some states compared to others but it is likely that some price variation occurs in every state. Note that if all resources of identical quality can be purchased at the same price by all units in state then this validity problem disappears.

The selection of the exact revenue or expenditure dollar measure can lead to validity problems. For example a case could be made that certain expenditures (or revenues that are spent on these expenditures) such as transportation or revenues such as categoricals for food service should be excluded from equity measures. If, for other reasons, these expenditures or revenues are included, this could lead to a validity problem.

There does not appear to be a problem of cardinality or the measurement of dollars by a single index for each educational unit when dollars are used as the treatment. However, when dollar measures are employed both problems across educational units such as districts. Research on these problems has not yet proceeded to the point where either of these problems can be quantified except there is some evidence that price variation can cause differences in the value of certain equity measures.[/]

The next criterion evaluates the treatment's effects on other treatments that are of concern. In this paper we do not review the extensive literature that attempts to assess the effects of more or less dollars per student in an educational unit. From the Coleman report[/] forward, researchers have tried to determine the effects of increased dollars and the most appropriate summary of the findings to date is "inconclusive." However, there has been some recent evidence that suggests that a linkage between educational spending and achievement orientation, verbal ability, years of completed education, and average wages or earnings may exist.[/*] That is, if a change to a more equitable distribution of dollars leads to more dollars for those who previously received fewer dollars of schooling inputs then we may have some reason to believe that other treatments such as schooling outputs and schooling impacts may also be affected. Since dollar measures of inputs (within states) are found in many instances to correlate with verbal ability, years of education, earnings, etc., it may be that changes in dollars of schooling inputs that are judged to be more equitable will lead to more equitable distributions of other treatments. But this conjecture is incompletely tested and, as indicated, insignificant effects are found in many studies. Therefore, while there is some evidence of the existence of a linkage between dollars of educational inputs and other treatments, the evidence is weak and the linkages should be considered possible but tenuous.

The fourth criterion is whether the states can control a treatment measured as dollars of schooling inputs. Since educational units such as school districts are creations of the states and since the states establish the structure and content of the financing system for primary and secondary education, there are almost no reasons why measures of dollars of schooling inputs are not consistent with the incentive aspect of the Federal legislation.

The final criterion of data availability is satisfied with mixed success but data on dollar inputs are more available than any other treatment we evaluate so relatively this treatment ranks high on availability. Surprisingly, although data on dollar inputs are collected by state related units such as Departments of Education, to our knowledge there is not a national data set of dollar inputs for all school districts that employs a set of common definitions. Therefore, at this time when the equity or equality of the state system is assessed, a data set of dollar inputs must be obtained from state sources, usually with state specific definitions that can vary considerably across states. It should be pointed out that a universal sample at the Federal level is expected to be implemented in the next few years. The Federal government does collect certain data for a national sample of school districts[/] but a national sample may not be appropriate for state specific assessments. Currently data are available at the state level but definitions may not be comparable nationally.

11. Price adjusted dollar inputs[1]

Since one of the major problems that emerges from a consideration of schooling input treatments measured as dollars is the problem of comparability across districts due to price variations, it is appropriate to determine whether we have available methodologies to adjust dollars for different purchasing power. Certain techniques have been developed to adjust dollars for prices[1*] so that we can evaluate price adjusted dollars as a treatment for our equity conception.

Based on our first criteria, conceptual desirability, price adjusted dollars should be preferable to dollar based measures. Price adjusted dollars potentially can measure resource inputs more accurately so that if price adjusted measures are clearly preferable on the other four criteria then they should be preferred.

However, research on price indices is in a developmental phase and some definite problems emerge in terms of measurement. First it appears that a "market basket" approach to price index construction is not appropriate. If an existing index is utilized, it may not capture schooling input price variation. This leads to a second approach, an examination of the schooling inputs themselves. But in this case, the measurement of price variation is difficult since this variation is caused by both supply and demand factors and conceptually we should only include supply factors that are not controllable by the school district in a price index. Interwoven in the supply-demand problem is the issue of input quality. A price index should compare inputs of

identical quality but this may be impossible if quality can not be observed and measured. Most recent research on price indices has utilized statistical econometric techniques to identify a supply function, and subsequently a price index, but it appears as though there are still significant measurement problems.

In terms of reliability, there is a good chance that any particular index includes some measurement error since the components of the model used to estimate the indices are measured with error and the estimated regressions do not explain all the variance. A more serious problem of validity may be applicable to the econometric price indices since they are highly dependent on a priori assumptions that are very difficult to test. Certain of the recent research efforts have compared alternative specifications of indices and found that the indices can vary substantially. Thus, we are faced with the question of which index, among many, to choose and whether the chosen index is more valid than no adjustment at all. Although we may believe that indices estimated with "reasonable" assumptions and with adequate data take us closer to resource measures than do dollar measures, we are not yet in a position to base these results on existing research. Thus, the issue of validity and reliability raise difficult but unanswerable questions.

For the most part, price indices have been applied to teacher's salaries or wages. However, there are other expenditure components that could be included in the input treatment, so we are faced with the task of estimating price indices for all components of inputs.

In order to measure the treatment in a single unit either unadjusted dollars can be added to price adjusted dollars or price adjustment techniques can be developed for these other components. However, for many of these other components the computation of an index may be more difficult than for teachers. The price adjusted measures are cardinal measures and presumably their use as the treatment increases the comparability of the treatment across educational units in a state compared to the use of unadjusted dollars. But this presumption stands untested; further research is definitely needed. Basically, we are at a stage where we have the ability to measure price adjusted dollars but we cannot conclusively evaluate the level of this ability.

The hypothesis that price adjusted dollars are linked to other possible treatments such as schooling outputs or schooling impacts has not been tested in the empirical literature. Most studies of the linkages between schooling inputs and schooling outputs or impacts have utilized either dollars or resources. There is the possibility that some of the observed linkage between unadjusted dollars and outcomes and impacts may be the result of price differences rather than real effects.[/] But there has not been a test of this directly so that, as we did for unadjusted dollars, we must again assume that the linkages may exist.

On the criteria of control, the adjustment of dollars for price effects does not appear to lessen the degree of control over the treatment that can be exercised by state policy. If states desire to alter the distribution of price adjusted dollars, they can include a price index in the state aid allocation system and

there is some precedent for this.[1]

The data availability criterion raises serious problems for the use of price adjusted dollars. The research work to date on price indices has primarily been exploratory and very few of the indices are fully developed so that they can be applied in a particular state for allocation decisions on either a state or federal level. Furthermore, the existing research has been carried out in a handful of states so that for most states, statistically derived price indices do not exist. If it turns out that price indices can be specified with the district as the unit of observation in the models, then the data may be available in most states to estimate a price index. If it is preferable to use individuals (i.e., teachers) as the unit of observation in the models, then most states will not have data available.

In either case it would be difficult to use price adjusted dollars in state or federal allocation mechanisms at this time until some of the measurement and data concerns expressed here are addressed. As research continues, we may find that certain simple indices are adequate proxies but this remains to be seen. In addition to the needed research on specification, questions such as whether a particular specification or set of specifications should be employed by each state or whether the choice of a price index is solely a state matter need to addressed.

iii. Resources and quality adjusted resources

The third way in which we can measure schooling inputs is by measuring the resources directly instead of using dollars or adjusting dollars to more accurately reflect the purchasing power.

Conceptually, equity on the input side generally refers to the resources available to the other children in the schools so that if the resource based measures satisfy the other criteria, they should be preferable to either dollars or price adjusted dollars in terms of what we desire to be equitable.

Resources can include the personnel involved in the schooling process such as teachers, aides, administrators, guidance counselors, and the non-personnel resources such as text books and supplies. Resources could also include part of the capital costs including buildings, equipment and furnishings. All of these resources presumably have an effect on schooling and could be treated as legitimate schooling inputs. Since we can attempt to measure resources with or without taking the quality of the resources into account we consider both of these possibilities in this part.

Resources can be utilized as a treatment in an equity conception only if we can measure them. We will assume the resources that are included in the treatment have been determined and that we are to assess the problems of measurement. There are a number of ways to measure resources, once identified, and these include counting each resource separately, counting resources separately but converting them to a common measure, counting separately but adjusting for quality and conversion to a common measure. For example, assume there are two resources: teachers and textbooks. We can count the number of teachers and books separately to yield two resource measures for each educational unit. Or we can determine the relationship in resource terms between teachers and books and convert them to a common measure.

Alternatively, we can count separately making adjustments for the fact that all teachers and books may not be the same in terms of the quality of the resource as a schooling input and, finally, we can try to take quality into account when we combine teachers and books.

While these four measurement techniques can be operationalized such that the reliability problems (i.e. measurement error) are minimized, serious validity problems are present if we move beyond the counting separately methodology. We have a desire for a single index for each educational unit, and therefore, the counting separately methodology is not appropriate. Since human resources form such a large percentage of schooling inputs it may be satisfactory to only measure these, but even here we must find a common denominator for teachers, teachers aides, administrators, etc. Once we try to combine personnel with other resources the validity of the technique becomes more highly questionable. Furthermore, quality differences among the resources are probably important and should be taken into account, but a valid methodology for measuring resource quality has not been developed. Thus, the validity problems associated with the development of resource indexes appear to be serious, although less serious if we measure one resource such as teachers or professional staff.

It does not follow from these validity questions that alternative resource based measures cannot be developed. Pugh et al. have used resource type measures to compare the interdistrict distribution of resources across states.[/] Pugh et al. use a measure of resources in which they count (for each district) the number of instructional staff members at each degree level and then multiply the number of instructional staff measures at each

degree level by the average salary nationally for that degree level. This represents the instructional resource component converted to dollars, and non-instructional expenditures are added to instructional resource dollars to produce their measure, "current expenditures with salaries controlled by degree level". Two assumptions in this measures are first, quality differences across degree levels are captured by national salary averages and second, other resources are equal to dollars.

Thus resource based treatments can be computed as treatments for use in an equity conception. We can construct a single measure with cardinal properties but the validity questions involved in combining the resources to a common index and measuring quality differences among particular inputs have not been answered satisfactorily.

Our next criterion is concerned with the linkages among resources and schooling outputs and impacts. This literature, which includes the educational production function studies, is vast and we will not attempt to summarize it here.[/] However, much of this literature, when viewed in its totality, presents a rather mixed picture. Most of the studies use some form of cognitive test score as the dependent variable and in only some of the cases do resource measures under the control of schools have statistically significant effects. It should also be noted that many of these studies use aggregated data at the school or district level while a number of more recent studies utilize student level data.[/*] While it is too soon to determine whether the inconclusive findings of the earlier studies are attributable to data and specification problems, some evidence that the linkages between resources and schooling outputs may exist and can be documented statistically. Thus, the conclusions

are not that different for resources than they were for dollars although considerably more production function research uses resource type variables.

The control by states over resources as opposed to dollars is somewhat less direct, but dollar control should be sufficient in most cases. The control is less direct since the choice of what particular set of resources to utilize is primarily a local decision yet control of total dollars at the district level can be attributed ultimately to the state. Thus, depending upon the way in which the resources are measured, in terms of formulating a single index and assessing quality, a state may be able to insur equity or equality of adjusted or unadjusted dollars, but this may not necessarily lead to the same degree of equity or equality of resources. For example, if the resource measure only includes a subset of potential resources, then this may create more difficult control problems since changes in dollars may not show up at all in resource measures. States would appear to have considerable control over resource, but not the same high degree of control tha exists for dollars.

The final question for resources measures is the availability of data. Again, most data on the resources should be available at the state level but it probably exists with considerable problems in definitional variability. While certain resource counts (i.e. professional staff per district) may be recorded at the state level, it is obvious that until quality distinctions are clarified, we can not assess the degree to which quality adjusted resource data are available even within districts. Again, certain

data pertaining to resources are available for a national sample of school districts, but this is not appropriate for state level analysis due to the sampling problems and the validity of the equality measures.

b. Schooling outputs

The second major category of treatments, schooling outputs, can be thought of as those qualities, characteristics, and skills that are developed through the schooling process.[/] Cognitive skills, often measured by achievement tests, are usually cited as the most important schooling output. However, the addition to these cognitive skills, certain affective qualities such as obedience to authority or punctuality are also transferred through the schooling process. Note that schooling outputs can be affected by a large number of factors other than schooling inputs including non-school factors such as learning in the home, student attitudes, and a student's native language, and school factors such as "efficiency" in the way in which the inputs are combined to produce outputs. Therefore, it is virtually impossible to "adjust" inputs for all the factors that are likely to lead to varying outputs. Thus, a conceptual preference for outputs is different from a preference for inputs as the treatment in an equity conception although the other four criteria must be considered along with the conceptual preference criterion.

There are a large number of potential measures and we will only discuss two different types in this part. First, we will consider the "traditional" measure of cognitive skills. Second, we

will discuss the use of behavioral rather than cognitive measures. The high school graduation rate and the college attendance rate could both be looked upon as behavioral measures of schooling's output.

As pointed out earlier in the report, equity of schooling's output does not have to be defined as equal schooling outputs. While equal levels of schooling outputs is one definition of of equity, at least two other possibilities exist. If a student's initial level of the output variable could be measured prior to entrance to the educational system, then equity could be defined as equal gains or increments for all students. Finally, if the inputs or resources required to obtain equal gains varies considerably over the population of students, equity could be defined as equal gains per dollar of resource for all students. This last possibility is more of an efficiency than an equity goal, nonetheless, equal gains per dollar of resource could conceivably be consistent with some people's equity conception. Furthermore, output equity could focus on the process by which the outputs are distributed rather than the output.

These possible equity notions are presented only to point out that the use of a schooling output as the treatment in a conception of equity need not imply equal outputs. As we examine the output measures on our remaining four criteria the selection of one of these possibilities is left open, since the choice does not play a key role in the evaluation.

i. Measures of cognitive skills-achievement scores

The study of achievement scores and their measurement has

roduced libraries full of information and debate so we can only hope to touch briefly on their use in an equity conception in this part.

The measurement of cognitive skills with achievement scores is an intricate part of almost all public school systems. The education profession seems to be reasonably that problems of reliability and validity in achievement testing are minimal. For example, Kerlinger states "(standardized achievement tests) are the products of a high degree of professional competence and skill in test-writing and, as such, are usually quite reliable and generally valid."/ However, validity in this case probably refers to internal validity; that is math achievement tests measure math achievement. External validity questions such as whether math achievement has an impact on experiences or behavior after schooling are not necessarily as straightforward to answer./* Furthermore, most people agree that achievement scores are only a partial measure of schooling outputs.

While we can conclude that, in general, reliable and internally valid achievement test scores can be developed, measurement in a conception of equity requires more specificity. For example, two issues that are addressed in a recent report that evaluates the use of achievement test scores to allocate Title I funds/** are the particular subjects that are appropriate for testing and whether a norm referenced or criterion referenced test should be employed. These two issues could be resolved so that a single comparable achievement measure results, but certain of the decisions leading to the development of the measure are likely to be controversial.

Thus, we are faced with the prospect of choosing among a number of achievement measures, each of which probably measures a somewhat different component of achievement. But once a particular test is chosen, comparability should follow. Our conclusion that the measurement problems associated with the use of achievement test scores can be overcome with existing measurement technology is based, in part, on the findings of the NIE study of Title I.[/]

Our second criterion is concerned with whether the measures of cognitive skills are linked to other possible treatments such as further education (i.e. college going behavior) or earnings. There have been a large number of studies that attempt to sort out the causes or determinants of further education and earnings. A review of these studies[/*] indicates that measures of "ability" have a statistically significant effect on further education and earnings but the interpretation of this effect in terms of the relative importance of ability compared to to other factors, especially in the case of income is not necessarily straightforward Furthermore, many of the "ability" measures used in these studies are not strictly achievement measures; usually they measure a combination of achievement and aptitude. None the less, there appears to be evidence, albeit limited, that combined aptitude and achievement measures are linked to other possible treatments, but the strength of the linkage is open to question.

The issue of whether or not the state and local educational system can control the treatment is the first of two criteria we examine specifically within the context of equity standards for Federal legislation. Acheivement levels or changes can be controll

ly partially by the local district and school since achievement is influenced by non-school factors and we currently do not have the knowledge necessary to adjust school factors to yield desired outputs. Control at the state level is even more difficult and removed since many of the decisions that impact achievement levels and change are not subject to state control under the existing system of school district organization. Even for the most far-reaching finance reform such as full state funding, it does not follow that the state has much control over achievement levels. Thus, the use of achievement levels does not appear to be an appropriate treatment from the standpoint of state control under the current system of organization. This is not meant to imply that there are no controls available to states to impact achievement scores in the school districts. But the control by the state of students' cognitive skills is significantly less direct than over dollars or resources when any of the suggested treatments are measured at the district level.

Our final criterion deals with data availability and, as the NIE study has pointed out, a national data base that can be utilized to assess the distribution of cognitive skills in the states does not now currently exist.[/] There is a national data base but, as was the case for national financial data, the data base is not designed to yield accurate state estimates. State administered tests are not appropriate either due to the differences among the states and the fact that some states do not administer tests. Therefore, data are not currently available to use achievement tests as the treatment in the equity conception for federal legislation.[/*]

ii. Behavioral measures of output - high school graduation and college attendance rates.

If an output measure is conceptually preferred as the treatment in a conception of equity, then an alternative to using a measure of cognitive skills is to measure the output in terms of the progression of students in the schooling process. Progress can be measured in terms of the average years of school completed, the percentage of students who graduate high school, or the percentage of students who are eligible to or actually continue their education.

Average years of school completed or high school graduation rates are similar measures in that they only consider behavior in the school system. College eligibility may differ from high school graduates in states where there are not institutions that accept all high school graduates. And college attendance depends on a number of additional factors including available resources and accessability in addition to a student's attitudes, ability, and program in high school. College attendance can be measured as the percentage of a given base year (say, ninth grade three years earlier) who continue on to college or the number of years of education, both secondary and post secondary, completed by a given base year.

There are obvious differences between measures of high school progress or completion and measures of college attendance. However in this part we focus on measures of high school progress since many of the issues are the same for both types of measures. But, in cases where there are marked differences, we will discuss the conclusions for college attendance measures as well.

There should not be any reliability related questions for the behavioral measures of schooling output. Similarly they can be represented as single cardinal measures. However, the most severe measurement related problem is validity. Does a measure of high school completion (or college attendance) computed across school districts in a state reflect the outputs of schooling? The basic problem is that it is difficult, if not impossible, to assess the output of schooling just by knowing that individuals graduate from a particular district. Comparability among districts is not assured since the output of high schools can vary substantially between and within districts and therefore measures of high school graduation do not control for quality. To a large degree graduation is defined by intra-district standards so that identical high school graduation rates may reflect largely different outputs across districts. That is, all high school graduates are considered identical in the computation of the output measure.

Therefore, while we can measure progress through the education system, the validity of such a measure of schooling outputs seems questionable since quality differences are ignored. These quality problems introduce considerable uncertainty if we desire to use the output measure comparatively across districts.

The linkages between behavioral measures of schooling completion and other possible treatments such as earnings are fairly well documented. In estimated earnings functions the variable that represents education has been consistently found to be significant. However, it should be noted that over 80% of ninth graders can be expected to graduate high school so that much of the variance in

the years of education variables in earning functions with recent data represents differences in post-secondary education.[1]

The ability of the state to control the behavioral output measures is similar to that for cognitive skill measures. The behavioral outputs are influenced by a large number of non-school factors and factors that are controlled by the school and district, but little control is exercised by the state. Again, states could introduce incentives for the districts to raise the behavioral output measures but this is not currently undertaken given the responsibilities of state versus local jurisdictions. Also, it is questionable to use a locally defined measure such as high school completion as an incentive at the state level since high school graduate rates can be achieved by altering the standards.

Finally, there is not to our knowledge regular sampling or population surveys of high school graduation rates. Data are available from census documents and other diverse surveys but there is not a yearly uniform coverage of school districts at the state level. Clearly, this type of data could be collected rather straightforwardly if a universal survey of school districts were implemented.

c. Schooling impacts

The final group of possible treatments for a conception of equity is school impacts which can be thought of as the effects of schooling on the larger society. Actually, the preference for schooling impacts is probably based on the treatments themselves;

Some people believe that income or satisfaction should be what is distributed equitably. As it turns out, we have reason to believe that education influences these treatments and furthermore, since education is often justified on the basis of the links to these treatments we label them schooling impacts. But it must be stressed that a wide range of other factors, in addition to education, have a profound influence on the treatments we have labelled schooling impacts.

The impact of schooling can be measured at the same time the schooling is produced, what is discussed below as current schooling impacts, or during the future after the education is produced, future schooling impacts. Obviously, it is extremely difficult to measure the impact of schooling so we rely upon reasonable proxies.

i. Current schooling impact

At any point in time, the focus of an equity conception could be the satisfaction that individuals receive from their daily activities. If satisfaction was measurable, it would certainly qualify as an appealing treatment in an equity conception. However, it may be impossible to measure satisfaction, per se, in any meaningful way. Yet, a large number of proxies have been developed by sociologists, psychologists and economists. In general sociologists have relied on status measures such as occupation and income, psychologists on measures of well-being often obtained through direct questions, and economists on utility, which is often based upon income. In this part we illustrate the measurement of a current schooling impact using a particular proxy for satisfaction although the issues

that are raised apply to the range of alternative satisfaction measures.

The formulation considered in this part is that of the economist where utility functions and social welfare functions are employed to measure satisfaction. The economist assumes that individuals strive to maximize their satisfaction or well-being where the object of their maximization is referred to as utility. If each individual strives to maximize his or her utility, the well-being or satisfaction of a group or society of individuals can be thought of as a function of the individual utilities. The relationship between individual and societal well-being is often termed a welfare function.

The assumptions, restrictions, and problems associated with the specification of utility and welfare functions are extremely complex and controversial.[/] However, it should be apparent that measures of individual and societal well being (i.e. utility and welfare) if available, could be desirable treatments in an equity conception.

With the important caveat that there are many assumptions behind any formulation and the understanding that there are an infinite number of potential utility and social welfare functions, it is worthwhile to consider one possible formulation that has been utilized within the context of school finance reform.

A recent formulation by Robert Inman[/*] includes specification of utility and welfare functions that can serve as an example of a treatment that represents current schooling impacts. The utility function specified by Inman is based upon two sources of utility: an individual's after tax income (Y) and current expenditures per

pupil (ED).[1] That is, an individual's utility, U, is some function of Y and ED and can be represented as follows:

$$U = f(Y_i, ED_i) ,$$

where the subscript i identifies each individual. Note we can use the family unit, instead of the individual, so that the family (f) utility function can be represented by $U_f = f(Y_f , ED_f)$.

Once we measure family utility, we can specify the treatment for the society as a welfare function, W, of the individual utilities as follows:

$$W = g (U_f) \text{ for all } f.$$

Thus, the treatment for this example would be the welfare derived from family utility.

These functions are conceptual as specified and clearly a large number of assumptions must be made before this formulation can be considered operational. We will assess some of these assumptions shortly when we consider the measurement criterion for this treatment; other assumptions will be explored later in the paper when we discuss equity criteria.

If specific utility and social welfare functions are utilized as a treatment in an equity conception, the various measurement issues all revolve around validity. Clearly, U_f and W can be specified to yield a single measure that can be used cardinally and compared across jurisdictions. But the formulation of such a measure raises a host of validity issues.

First, questions can be raised about the variables included in the utility function. If people do receive utility only from education and income, are per pupil expenditures and annual income the most appropriate variables? The omission of other public

expenditures as well as satisfaction derived from activities not adequately reflected by income may not be valid. Second, there is the question of the functional form that is used to combine the variables to yield a utility index. There are certain properties of welfare functions that are considered "desirable" such as symmetry and concavity yet there are an infinite number of utility functions that meet these criteria. A methodology has not yet been derived to test the validity of alternative specifications.

A third and important validity question arises when we combine individual utilities to yield a social welfare function, W. How do we determine the appropriate specification of W? Can we use utility measures as though they are cardinal measures? Furthermore, are we justified in comparing utilities across individuals, families or districts? Each of these questions lead to serious problems that may be, under certain assumptions, impossible to resolve.

Although welfare and utility based measures can be calculated as shown by Inman, there are serious unanswered validity questions that remain. However, two additional points should be made. First, in certain instances, some of the validity problems can be partially obviated through the use of sensitivity analysis. While this holds true for validity problems for any treatment, it is relevant here since Inman has shown the degree to which certain school finance policy reforms are senstivie to alternative welfare functions and equity criteria, among other things. Second, note that in a utility function of the form $U = f(ED_i, Y_i)$, when the impact of income, Y_i, is zero, utility is only a function of per pupil expenditures which, in some ways, is analogous tc the use of schooling

puts. Thus, although we may raise serious validity questions concerning a utility function approach, these questions are not necessarily answered if we opt for a different treatment.

While the validity issue appears to be paramount, the questions of control and data availability should also be addressed. Without specifying a particular utility function it is fairly clear that a measure of income is likely to be included in almost any utility specification. But income is not a variable that can be controlled by the individual states and, therefore, it is not a proper variable for an incentive system. The second variable used in the example, educational expenditures, can be controlled but when utility is the treatment, control of income is desirable.

Finally, income data by school district are not readily available. A particular census project was required for 1970 information to match income data to school districts. While a few states may have the capability to calculate district level income figures from state income tax data, income data by school districts are usually non-existent.

ii. Future schooling impact

An argument can be made that the impacts of schooling should be measured over an individual's lifetime, not at the time the schooling takes place. Furthermore, lifetime satisfaction, utility, or income can be plausible treatments in an equity conception, regardless of their relationship to schooling. Thus, in this part we consider the utilization of a future schooling impact as the treatment in an equity conception.

Utility, satisfaction, status, income and perhaps other concepts could be candidates for a treatment in a lifetime concept of equity. However, for this discussion we limit ourselves to income with the assumption that the problems associated with this treatment are present for the others as well. In fact, the difficulties identified for income should probably be multiplied substantially for the other alternative treatments.

For this discussion, it is assumed that a desirable treatment is the lifetime income that will accrue to individuals who are currently completing their education. The first question is whether or not this can be measured so that it satisfies the measurement criteria.

Since future income is, by definition, unobserved, an estimation procedure must be devised to measure the treatment. Given the large body of research on earnings functions, it is likely that a procedure could be devised so that the distribution of lifetime earnings of a particular cohort in the education system could be estimated. However, since earnings are dependent on a number of variables that occur after high school graduation and the earnings functions themselves typically explain less than twenty five percent of the variance in earnings, it is unlikely that the estimation procedure is likely to lead to a reliable or valid estimate of future lifetime earnings. Furthermore, the earnings functions themselves are estimated using historical or current data and the estimated parameters themselves may change in the future. While the measures would be cardinal and, depending upon the estimation procedure and price level adjustments, comparable, the reliability and validity

problems appear to be extremely serious.

Even with these difficulties, it would be possible to estimate future earnings distributions for cohorts in each state and some of the measurement problems may be negated to some degree if the distribution are compared with one another, rather than with the actual distribution that occurs. But this assumes that the measurement problems will affect each estimate identically and each estimate will bear the same relationship to the actual. Both of these assumptions are closer to leaps of faith rather than sound measurement assumptions given our current understanding of earnings functions and the variables that are necessary for prediction.

As was the case with current income, it seems unreasonable, given the current allocation of fiscal responsibilities, that state governments could control future lifetime incomes to any significant degree. They may be able to influence some of what we believe to be the determinants of income but this would fall far short of any meaningful control over the distribution of lifetime income.

Finally, it is doubtful that the data are available, at a state level, to estimate distributions of future lifetime income. Income data are not generally available and the data requirements for earnings predictions are even more demanding.

D. Criteria

1. Children and Inputs, Outpus, or Impacts

The criterion one uses to evaluate equity is one of the most important components of a conception because of the differences in judgements on equity that can be produced and because many unresolved philosophical problems are embedded in the choice of a criterion. For ease of exposition, this section is subdivided into two parts. First, different concepts of equity using identical children as a unit of analysis are presented, then alternative equity criteria for non-identical children. Households as a unit of analysis are treated following these sections.

a. Identical Children

i. Introduction

If children are all the same with respect to variables such as the cost of educating them, and a treatment has been specified, then the problem an equity criterion needs to resolve is how to evaluate differing degrees of horizontal equity. Horizontal equtiy is concerned with the equal treatment of equals. If children are identical, they are all equal and perfect equity is synonomous with perfect equity. The equity criterion's task is to measure the degree to which the distribution differs from equality.

The problems of measuing the inequity of a distribution can be approached by specifying the characteristics of a social welfare function (SWF) that embodies a certain equity conception and subsequently identifying a summary number that will rank distributions consistently with that SWF. This is the first approach pursued. In Part ii the

concept of a social welfare function is first defined. Then the use of a specific class of SWF's in an equity index is described. Part iii presents a measure of equity that is based on a SWF that is less restrictively defined than the one utilized in Part ii's index, and Part iv discusses issues of the level of measurement.

Unlike the first approach that is closely linked to specific SWF's, the second approach is content to identify a number of value judgments inherent in an equity criterion (Part v). These value judgments are then paired with specific summary measures of equity. In some cases the results of the two approaches (SWF and value judgments) overlap. However, there are a significant number of widely used summary measures that cannot be fit into the SWF format and for these measures the value judgment approach is more appropriate.[/]

ii. Social Welfare Functions and Atkinson's Index of Equity

A social welfare function (SWF) is a rule for assigning a number to every possible state of the world such that the numbers reflect preferences for the different states. The characteristics of a chosen rule embody values about various dimensions of preferredness. The use of SWF's in equity evaluation often involve a transformation of the SWF into an index (or measure) that can be given an intuitive interpretation.[/*] The transformation retains the basic values contained in the SWF and therefore the characteristics of the index will depend on the value judgments expressed by the SWF on which it is biased.

There are three characteristics of indexes that are derived from the study of SWF's that are important to the understanding of the equity conception implied by the index.[/**] The first characteristic is

referred to as the degree of inequality aversion. If an index displays constant relative inequality aversion, then when every treatment level in a distribution is increased by an equal proportion (for example, by 10%), the index will not change in value. The index could also display constant absolute inequality aversion if equal absolute additions to each treatment level leave the value of the index unchanged. Equality aversion characteristics do not keep the mean value of a distribution the same when comparisons of the index are made.

A second important characteristic of an index is based on the presence or absence of what is called the Pigou-Dalton condition. The presence of this condition means that transfers of a constant dollar amount from levels of treatment higher in a distribution to levels of treatment lower in the distribution always increase equity according to the index, as long as the original order of members of the distribution remains unchanged. Such transfers do not affect the distribution's mean value.

The third characteristic of the index is the importance it assigns to the treatment levels at different places in the distribution. The value of an index can be determined by nearly equal weights given to all treatment levels or different treatment levels.

A commonly used SWF-based equity index has been devised by Anthony B. Atkinson.[/] Atkinson's index requires that a functional form for the SWF be specified. The index uses the SWF to determine what Atkinson labels the equally distributed level of treatment, or

$_{EDE}$. Y_{EDE} is the per capita treatment that if equally distributed would result in the same level of Social Welfare as the existing distribution of (non-equal) treatments. His index of equity is:

$$I = 1 - \frac{Y_{EDE}}{\mu}$$

or 1 minus the ratio of the equally distributed equivalent income to the existing mean income. Y_{EDE} is less than or equal to μ for most specifications of the SWF, so I ranges from 0 (complete equality) to 1 (complete inequality). Atkinson has a convenient interpretation for I as applied to income as a treatment: "If I = 0.3, for example, it allows us to say that if incomes were equally distributed, then we should need only 70% of the present national income to achieve the same level of social welfare (according to the particular social welfare function)".

A SWF that is commonly used in Atkinson's index is one that results in an index with constant relative inequality aversion. The SWF is:

$$SWF = \sum_{i=1}^{n} \left[A + B \frac{y_i^{1-E}}{1-E} \right] \left[f(y_i) \right]$$

$E \neq 1$
$E \geq 0$

or

$$SWF = \sum_{i=1}^{n} \left[A + B \log y_i \right] \left[f(y_i) \right]$$

$E = 1$

where:

y_i = level of treatment per child i

n = number of unique treatments in the distribution

$f(y_i)$ = relative frequency of y_i.

A = any constant

B = constant greater than zero.

E = constant greater than zero.

The crucial parameter is 'E' because the choice of a value for this parameter can vastly change the value judgments inherent in the SWF and consequently in Atkinson's index. The larger the value of E, the more concern is shown for the lower end of the distribution. The limit as E goes to infinity is an index that depends solely on the lowest valued treatment in a distribution. As explained in Appendix 1, the index based on E equal to infinity is a mathematical representation of John Rawls' principle of justice.

Atkinson's index based on the above specified SWF does not always meet the Pigou-Dalton requirement. In particular when E is equal to infinity it is not met.

The mathematical form of Atkinson's index when the SWF just discussed is used is equal to the following expression:

$$I = 1 - \left[\sum_{i=1}^{n} \left(\frac{y_i}{\mu} \right)^{1-E} f(y_i) \right]^{\frac{1}{1-E}}$$

$$E \geq 0$$

Where each symbol is defined the same as it was for the SWF. The index automatically displays constant relative inequality aversion, may or may not staisfy the Pigou-Dalton condition, and weights the low end of the distribution more (is more egalitarian) the larger

is the chosen value of E. Atkinson's index can be used cardinally or ordinally. It is an excellent measure except that the specification of a value for E is a very demanding requirement for its use because the evaluator is required to exactly state her/his degree of egalitarianism.

iii. Less Constraining Forms of Social Welfare Functions and Measures of Equity.

It is not always necessary to state the SWF in the relatively constraining form of a specific mathematical function in order to derive a summary measure of equity. Amartya Sen and others[/] have been able to demonstrate that a class of SWF's that can be described by relatively few characteristics will rank distributions consistently with a commonly used summary measure. Specifically if the SWF is symmetric and strictly quasi-concave and if the distributions being compared have either the same number of members and the same total level of treatments or different numbers of members and the same mean level of treatments then the gini coefficients from non-intersecting Lorenz curves will rank the distributions consistently with the SWF. A symmetric SWF is one that is invariant with respect to which member of the group is in each position of the distribution. Only the distribution itself is important, not who is at the top or bottom.[/*] The characteristic of "strictly quasi-concave" is a slightly less restrictive mathematical requirement than concavity, but its interpretation for the purposes of this paper is identical to concavity. Concavity means that as the treatment level of any one member of the group increases, ceteris paribus on everyone else's treatment level, the SWF function increases, but a diminishing rate.[/**]

The correspondence between non-intersecting Lorenz curves and symmetric and strictly quasi-concave SWF's is appealing because of the generality of the SWF and the limited number of specific value judgments built-in. Unfortunately the requirement of non-intersecting Lorenz curves and the requirements on population size comparability and/or mean treatment comparability severely limit strict application in practice. Given the kinds of distributions one inevitably wants to compare, it is almost always necessary to revert back to the specification of a mathematical form for the SWF or to the less elegant approach of stating value judgments attached to the summary measure without specifying the underlying SWF.

iv. The Level of Measurement Issue

The issue of the level of measurement inherent in the SWF's and their summary measures has heretofore been neglected. An important distinction can be made between ordinal and cardinal measurements. Both types provide a ranking from worst to best, with the possibility of ties, of all distributions being considered. Ordinality goes no further than this. Numbers assigned to levels of a SWF or a SWF-based index serve only to position each distribution with respect to the other and any positive monotonic transformation serves equally well. Cardinality on the other hand provides information on the magnitude of the difference between distributions as well as their positions. Cardinal numbers can only undergo a positive linear transformation and still retain their quality of distinguishing degrees of differences.[/] A cardinal measure can always be used ordinally, but an ordinal measure can

not be used cardinally. Atkinson's index is cardinal and can therefore be used to make quantitative statements about differences in equity among distributions. The Lorenz curves (gini coefficients) are only ordinally consistent with symmetric and strictly quasi-concave SWF's and therefore when thye are used to represent SWF's, they can only be used to rank distributions. The measures presented in the following sections are representations of combinations of value judgments. They are used in practice both ordinally and cardinally.

v. Value Judgments and Measures of Equity

The second approach to the establishment of equity criteria for identical children is not so closely linked to specific SWF formulations. Instead the second approach identifies a series of value judgments on equity that are implicitly expressed when several commonly used summary measures are employed.[1/] Some of these value judgments can be translated into SWF-based characteristics, such as the Pigou-Dalton condition or inequality aversion and these correspondences will be noted where thye are applicable.

The value judgments are formulated in terms of questions and these are displayed in Table 2. The questions in Table 2 are posed assuming that dollars are the chosen treatment and children are the unit of analysis, but of course, other treatments and units could be used.

The first question asks whether all children are included in the measure. Certain measures focus only on children at particular points in the distribution while other measures use all children to compute the equity measure.

TABLE 2

VALUE JUDGMENTS FOR EQUITY MEASURES
FOR IDENTICAL CHILDREN

1. Are all children taken into account in the equity measure?
2. Does the equity measure always show an improvement when dollars are transferred from one child to another that is lower in the distribution and both children are located on the same side of the mean?
3. Does the equity measure always show an improvement when dollars are transferred from one child to another that is lower in the distribution and both children are located on the same side of the median?
4. Does the equity measure always show an improvement when dollars are transferred from one child above the mean to another that is below the mean?
5. Does the equity measure always show an improvement when dollars are transferred from one child above the median to another that is below the median?
6. Does the equity measure always show an improvement when a constant amount of dollars is added to each unit?
7. Does the equity measure always show an improvement when the total dollars of each unit are increased by a proportional amount?
8. Does the equity measure record dollar changes at different levels of the distribution in the same way?
9. Is the mean level used as a basis of comparison?
10. Is the median level used as a basis of comparison?
11. Are all levels compared to one another as the basis of comparison?

A second set of value judgments is represented by questions two through five. Some people may believe that an equity measure should show an improvement if resources are transferred from a child higher in the distribution to one lower in the distribution and, therefore, affirmative answers to these questions would be desirable. Each measure considered is sensitive to certain kinds of transfers, but not others, and these four questions are posed to distinguish among different kinds of transfers. Note that the transfers described here do not change the mean of the distribution. Also note that the Pigou-Dalton condition is met if all four questions (2-5) are answered affirmatively.

A third set of value judgments is concerned with the over-all level of the distribution as represented by the mean of the distribution. The distributions that will be compared will usually have different mean values for their outcomes and the measures incorporate the mean level differently. Questions six and seven illustrate two ways in which the mean level can be taken into account. If question six is answered affirmatively, the measure possesses decreasing absolute inequality aversion. If question seven is answered affirmatively, the measure possesses decreasing relative inequality aversion. These characteristics are ones applied to SWF based equity indexes as well.

Question eight deals with the weighting of movements toward or away from equity when the movements occur at different points in the distribution. More specifically, some of the measures incorporate the belief that changes for children at the low end of the

distribution should somehow be taken into account to a greater degree than comparable changes at the high end of the distribution. Note that the answer to question eight is no when certain children are excluded from the measure (i.e., when the answer to question one is no) so that question eight is significant when question one is answered affirmatively.

The final three questions, nine through eleven, are concerned with the standard of comparison used in the equity measure. The mean or median is used in most measures, although some compare among all units.

The measures whose value judgments are specified are: the range, the restricted range, the Federal range ratio, the percentage of students within X% of the mean, the relative mean deviation, the permissible variance, the variance, the coefficient of variation, the standard deviation of logarithms, the gini coefficient, and Atkinson's index of equity using the previously specified SWF.[/] These measures are chosen as a very inclusive list of commonly used measures.[/*] Atkinson's measure is included in the list to increase the comprehensiveness of the approach.[/**] Figure 1 illustrates the relationship between the value judgments and the summary measures by providing an answer to each of the value judgment questions listed in Table 2.

In future sections of the paper, simplified models of the distribution of education treatments are presented. Several of these models provide numerical illustrations of how the use of a measure can influence evaluations of equity. Examples of conflicts in the value judgments are left until these later sections.

FIGURE 1

ANSWERS TO VALUE JUDGMENT QUESTIONS
FOR ELEVEN EQUITY MEASURES
FOR IDENTICAL CHILDREN

	EQUALITY MEASURES										
VALUE JUDGMENTS*	Range	Restricted Range	Federal Range Ratio	Relative Mean Deviation	Permissible Variance	Variance	Coefficient of Variation	Standard Deviation of Logarithms	Gini Coefficient	Percentage of Students Within X% of Mean	Atkinson's Index. E>0
1. All children taken into account?	No	No	No	Yes	No	Yes	Yes	Yes	Yes	No	Yes
2. Improvement for transfers on one side of the mean?	No	No	No	No	No	Yes	Yes	Yes**	Yes	No	Yes
3. Improvement for transfers on one side of the median?	No	No	No	No	No	Yes	Yes	Yes**	Yes	No	Yes
4. Improvement for transfers that cross mean?	No	No	No	Yes	No	Yes	Yes	Yes	Yes	No	Yes
5. Improvement for transfers that cross median?	No	No	No	No	Yes	Yes	Yes	Yes	Yes	No	Yes
6. Sensitive to equal additions?	No	No	Yes	Yes	Yes	No	Yes	Yes	Yes	Yes	Yes
7. Sensitive to equal percentage increase?	Yes	Yes	No	No	No	Yes	No	No	No	No	No
8. Changes at different levels recorded identically?	No	No	No	No	No	Yes	Yes	No	No	No	No
9. Mean for comparison?	No	No	No	Yes	No	Yes	Yes	Yes	No	Yes	Yes
10. Median for comparison?	No	No	No	No	Yes	No	No	No	No	No	No
11. All levels for comparison?	No	No	No	No	No	No	No	No	Yes	No	No

*For a more complete description of the value judgments, see Table 2 and text.
**At very high levels in the distribution, the answer may be no.

b. Non-Identical Children

i. Introduction

Children differ for a number of reasons as outlined in a previous section on children as a group (Section B2). If the differences are legitimate ones, such as physical handicaps or district resource costs, then the equity criteria need to go beyond the measurement of horizontal equity. Vertical equity, or unequal treatment of unequals, must also be incorporated. If differences are illegitimate, as is often the case for race, sex and property values, then the horizontal equity measures outlined for identical children can be used alone or they can be used with a bivariate measure in a way that will be described in section b that follows.

The measurement of vertical equity for legitimately differentiated children can be thought of as the measurement of the combination of two or more distributions, one of which is a distribution of treatments per child and the other(s) of which is the distribution(s) of differentiating characteristics of the children. The combination can be effected in two different ways. One way is by means of the univariate statistics described in the identical children section where each child's treatment and characteristic are combined into a single variable. The equity of the distribution of the newly created combined variable is then evaluated by means of a univariate statistical measure. As a brief example, vocationally educated children might be given a weight of 2 while all other children are weighted 1. If vocational education is the only legitimate difference among children, a univariate treatment measure of treatment per weighted child can be used to evaluate equity.

A second way to combine two or more distributions in an equity measure is by means of bivariate or multivariate statistics which are designed to summarize relations among distributions. For example, the equity of expenditures per child in relationship to a child's reading ability might be measured by a correlation, slope coefficient or elasticity from a bivariate regression.

This part of the paper discusses univariate measures for legitimate differences in Section ii and bivariate measures in Section iii. Appendix 1 contains some alternative univariate specifications not considered in Section ii.

ii. Univariate Measures

In order to utilize a univariate measure for legitimately differentiated children, each child's treatment and characteristics must first be combined into a single variable. The construction of the single variable can be accomplished by either weighting children according to a scale that represents the differences among them or by indexing the treatment by an index that makes the value per unweighted child have equivalent meaning across all children.

The first option, weighted children, can be illustrated by a hypothetical example. Suppose that the evaluators think that there are legitimate differences among the following four groups of children: K-6 handicapped; 7-12 handicapped; K-6 non-handicapped; 7-12 non-handicapped. Also suppose that the differences are quantified in such a way that handicapped are weighted three times non-handicapped, K-6 are weighted twice 7-12 and the weights are used multiplicatively. The weights assigned to children by group would be as follows:

Child's group	Weight
K-6 handicapped	6
7-12 handicapped	3
K-6 non-handicapped	2
7-12 non-handicapped	1

If each child were multiplied by the appropriate weight and treatment per weighted child were calculated, then the univariate equity measures presented for identical children could be applied as equity evaluators.

There are several important characteristics of the use of a univariate measure that should be noted. First the quantification of the weights represents a value judgment both about which children's characteristics are legitimate and about how different the children are. In the previous example, reading disability is not specially weighted, so implicitly such differences are not important. Also there may be differences of opinion about the appropriate weights to assign to handicapped or to elementary school children and these differences are value judgments. Even if the weights are determined "objectively" on the basis of cost per unit of constant quality education (or any other standard) there is still a value judgment in the choice of the basis for the objectivity.

The second characteristic of the univariate measure for legitimately different children is that it measures both vertical and horizontal dimensions of equity. The assignment of a specific weight to different children takes care of the vertical equity problem. The calculation of the univariate measure implies that the weighted children should be subject to horizontal equity or be treated equally. As will be seen in the next section, the

bivariate and multivariate measures are only concerned with vertical equity, so the weighted univariate measure might be considered more comprehensive.

Assigning weights to different children is one of the two options available for combining treatments and children's characteristics into one varaible. The second option is to transform the treatment so that it is equivalently defined for all children. The transformation could be accomplished by an index that represents differences in the value of a set level of treatment for different children. For example, suppose children are legitimately differentiated according to the price of a unit of educational resources in their school district. A price index could be devised for each child and the treatment received by each child could be divided by the index. Then the univariate measure would be calculated for price adjusted treatments per unweighted child. As with the weighted children option, the indexed treatment exhibits a number of characteristics. First the choice of indexes to calculate is a value judgment about which differences are legitimate. Second the value of the index may be a judgment. In the case of a price index, the value is supposed to objectively measure price differentials. However, there are likely to be judgments involved in determining which causes of price differentials are allowed to count.[/] For other indexes, for example ones based on children's reading ability, the judgments inherent in choosing specific values for the index are likely to be more transparent than for a price index. Finally, the transformed treatment option, like the weighted children option, combines vertical and horizontal equity dimensions in a single measure.

iii. Bivariate and Multivariate Measures

a) Regression Statistics: Correlation, Slopes, Elasticities

Bivariate and Multivariate measures summarize the relationship between two or more distributions without the necessity of combining two variables into one. The available measures are the bivariate correlation, the slope and the elasticity.[/] The slope and the elasticity can be calculated in a number of alternative ways, depending on the functional form of the regression equation from which they are derived. The simple, or bivariate, regression is an estimation of the following kind of relationship between a dependent variable, Y, and an independent variable, X:

$$Y = b_o + b_1 X + \text{Error}$$

The bivariate slope is the estimate of b_1. The bivariate elasticity, calculated at the means, is $(b_1 \frac{\bar{X}}{\bar{Y}})$. The bivariate correlation coefficient is $r = b_1 \frac{S_x}{S_y}$ where S_x and S_y are the standard deviations of X and Y respectively.[/*]

The regression equation may also be estimated as a polynomial of any degree. As an example, a quadratic regression would be the following form:

$$Y = b_o + b_1 X + b_2 X^2 + \text{Error}$$

The slope would be $(b_1 + 2b_2 \bar{X})$ and the elasticity would be $(b_1 + 2b_2 \bar{X})(\frac{\bar{X}}{\bar{Y}})$.

Finally, the dependent and independent variables in the bivariate regression may be transformed into their logarithmic counterparts as follows:

$$\ln Y = b_o + b_1 \ln X + \text{Error}$$

In such a regression b_1 is the elasticity, which is constant or does not vary throughout the domain of X and the range of Y.

The rest of Section a will concentrate on some general differences between the bivariate and multivariate measures. Section b will discuss more specific interpretations of the statistics as they relate to equity and Section c will present a series of value judgments that can be used to distinguish among the alternative measures.

The bivariate correlation coefficient, r , indicates the strength of the linear relationship or the goodness of fit between Y and X. A value of $\pm$ 1 indicates a perfect fit and a value of 0 indicates no relationship. The slope from any of the functional forms indicates the relationship between a unit of change in X and the consequent change in Y and is therefore more informative than the correlation coefficient, r, if one wishes to predict values of Y at different levels of X. The elasticity standardizes the slope by representing the percentage change in Y that is related to a 1 percent change in X. The elasticity can either vary in value depending upon the original levels of X and Y or it can be a constant. If it varies, then the means of X and Y are conventionally chosen as the values for which the elasticity is calculated. Table 3, which is discussed in Section c summarizes the alternative bivariate and multivariate statistics.

b) Interpretation of Bivariate and Multivariate Measures As Equity Criteria

There are two characteristics of bivariate and multivariate measures that are important to their use as equity criteria. First

the measure's interpretation will differ depending on whether the children's differences are illegitimate or legitimate. As an example, suppose expenditures per child are used as a treatment and district property value per child is used as an illegitimate differentiating characteristic. The measures indicate equity when the correlation coefficient, the slope, and the elasticity are all equal to zero. The more highly positive each of the three is, the greater is the level of inequity. The meaning for equity of negative values is unclear because negative values indicate that higher property wealth per child is associated with lower expenditures per child and this negative relationship may be considered by some evaluators just as inequitable as a positive correlation.

A legitimate difference among children requires a different interpretation of the measures. Suppose expenditures per child is once again used as a treatment and a dichotomous representation of handicapped is used as a legitimate difference, where a value of 1 indicates handicapped and a value of 0 indicates non-handicapped. Then positive values of the correlation coefficient, the slope and the elasticity would represent equitable distributions and zero or negative values would represent inequitable distributions.

The second characteristic of the bivariate and multivariate statistics used as equity criteria is that they do not measure horizontal equity. When they are used with an illegitimate difference such as property wealth per child, they can be interpreted as measuring the absence of bias or discrimination. An alternative interpretation for illegitimate differences is that the statistics measure the presence or absence of equitable or unbiased opportunity. In either case horizontal equity is not measured and unless

unbiasedness with respect to wealth is all that is required for equity, the univariate measures for identical children must be used to measure horizontal equity. In other words, two measures of equity, one for opportunity and one for horizontal equity may be required.

When the bivariate and multivariate statistics are used with legitimate differences, such as presence or absence of a handicap, only vertical equity is measured. Once again, if horizontal equity is also important, the univariate statistics will have to be used in addition to the bivariate ones. Although it may not be as elegant to have vertical and horizontal equity measured separately rather than together in one measure, there is one advantage to the bivariate representation of vertical equity. The vertical equity notions need not be as precise as when weighting of children or indexing of treatments are used in the univariate statistics to measure vertical equity. The evaluator need at first only specify the sign, not the precise numerical value, for the bivariate and multivariate statistics. The regression will depict the existing relationship and the sign of the relationship. The disadvantage of the regression representations of vertical equity is that the horizontal equity measure used in conjunction with it will not incorporate the vertical equity notions.

c) Value Judgments and Bivariate and Multivariate Measures of Equity

There are a number of value judgments that are implicitly expressed when each of the bivariate and multivariate statistics is used to measure equity. The value judgments are similar to the ones that were outlined for the univariate measures in Section b. v., Table 2 and Figure 1. Each value judgment is again formulated

in terms of questions and these questions are displayed in Table 3. The questions are posed assuming that the independent variable is an illegitimate difference such as property value per child and the group is children. Equity is therefore improved when the statistics decrease in value. We assume in the table that negative values are more equitable rather than uncertain in interpretation.

The first question asks whether all children are included in the measure. The answer is yes for all the measures, but the question is included to provide consistency with Table 2 and so that univariate and bivariate meausres can be compared.

The second question asks about an improvement in equity when a mean preserving transfer of the treatment (or dependent variable) is made from a child high in the distribution to one that is lower. Note that the independent variable, i.e. property value per child, is assumed to remain constant for this question as well as for numbers 3 and 4. The second question is similar to the Pigou-Dalton condition for univariate statistics.

The third and fourth questions ask about the analogue of absolute and relative inequality aversion for bivariate and multivariate statistics. The value judgments expressed in these two questions are concerned with the overall level of the distribution of the treatment as represented by its mean. The distributions of treatments that are compared will have different mean values.

Table 3

VALUE JUDGMENTS FOR EQUITY MEASURES FOR NON-IDENTICAL CHILDREN

1. Are all children taken into account?
2. Does the measure always show an improvement when a set quantity of treatment (dependent variable) is transferred from one child to another who is lower in the distribution?
3. Does the measure always show a change when each child's treatment (dependent variable) is increased by a constant amount?
4. Does the measure always show a change when each child's treatment (dependent variable) is increased by an equal percentage?
5. Does the measure always show a change when each child's characteristic (independent variable) is increased by a constant amount?
6. Does the measure always show a change when each child's characteristic (independent variable) is increased by an equal percentage?

FIGURE 2

ANSWERS TO VALUE JUDGMENT QUESTIONS FOR SIX EQUITY MEASURES FOR NON-IDENTICAL CHILDREN

VALUE JUDGMENTS*	Simple Correlation (r)	Simple Regression Slope (b_1)	Simple Regression Elasticity ($b_1 \frac{\bar{W}}{\bar{E}}$)	Constant Elasticity ($\frac{d \ln E}{d \ln W}$)	Quadratic and Higher Order Regressions Slope ($b_1 + 2b_2 W^2$)	Quadratic and Higher Order Regressions Elasticity ($b_1 + 2b_2 \bar{W}) \frac{\bar{W}}{\bar{E}}$)
Are all children taken into account?	Yes	Yes	Yes	Yes	Yes	Yes
Improvement for all mean preserving transfers in dependent variable?	Not Necessarily	Yes	Yes	Not Necessarily	Not Necessarily	Not Necessarily
Sensitive to equal additions in dependent variable?	No	No	Yes	Yes	No	Yes
Sensitive to equal percentage increases in dependent variable?	No	Yes	No	No	Yes	No
Sensitive to equal additions in independent variable?	No	No	Yes	Yes	No	Yes
Sensitive to equal percentage increases in independent variable?	No	Yes	No	No	Yes	No

r a more complete description of the value judgments, see Table 3. For exposition purposes it is assumed that E is a treatment (dependent variable) and W is the children's difference (independent variable).

Questions 5 and 6 are concerned with the behavior of the measures when the independent variable (child's characteristic or property value per child in our examples) is increased by a constant absolute or percentage amount. These are important questions when the independent variable is property value per child. States assess property at different percentages of market value ranging from below 33% to 100%. When cross-state comparisons (or intertemporal comparisons for one state) are made on bivariate measures of the relationship between expenditures per child and wealth per child it is extremely important to understand which measures will be affected by the differences in assessment ratios (percentage changes in the independent variable).

Figure 2 illustrates the relationship between the value judgments and the statistics by providing an answer to each of the value judgment questions listed in Table 3. A numerical example of the first four value judgment differences illustrated in Table 3 will be presented in the section of the paper on models of school districts.

2. Households and Total Goods and Services

a. Introduction

It was argued in a previous section (B3) that the only logical treatment to apply to the household as a unit of analysis is the total amount of goods and services, or total resources, available per unit of time. In this section, two ways are chosen to conceptualize total resources, both of them in a current or short run time period (for example, a year). The possibility of total life-time resource distribution is not considered at this point for the practical reason that the state of the art in measurement does not allow a good empirical measure of life-time resources.

The two concepts of resources used are total resources and changes in total resources. The use of total resources requires that the level of education services and the level of all other goods and services available for consumption by a household in a given year enter the equity criteria. The use of changes in total resources as a treatment means that net additions to resources (services minus payments for those services) enter the equity criteria. The changes in resource concept may also require a reference to total resources before the change.

Two ex post criteria are presented to evaluate equity in the distribution of total resources (or changes in total resources) among households. Following the presentation of these two ex post criteria, one ex ante criterion is discussed. The first ex post criterion is an application of Atkinson's equity index to households and involves the specification of a utility function that combines education and other goods and services as well as a social welfare

function that weights the derived utility levels. The second ex post criterion is a modification of the traditional tax equity principles of ability-to-pay. The ex ante criterion is one that relates the tax price of education to house values. Each of the criteria is presented in a separate section.

b. Atkinson's Equity Index Applied to Households And Total Resources

The distribution of total resources among households can be evaluated by means of a social welfare function and its transformation into Atkinson's index of equity. The steps necessary to derive Atkinson's measure for households are outlined below and the places where value judgments enter are highlighted.[/]

The first step is the derivation of an argument for the SWF that combines household education services and other goods and services into a total resource measure. The combination can be achieved by the specification of the argument as household utility, where utility is a function of education services (ED) and all other goods and services (Y). The measurement of ED and Y and the specification of the parameters of the utility function are important. ED is usually measured as the dollar flow of educational resources per year to the hourshold. Conceptually childless households could receive dollar flows via external benefits, but measurement of the magnitude of the externalitites would be difficult. Y is generally measured as after-education household income on the assumption that this income represents the quantity of all other goods and services available to the family.[/*] In general ED and Y are combined by a utility function $U_i[ED_i, Y_i]$ where i represents each household. This generalized utility function must be given a

specific functional form in order to eventually derive an index of equity. A common form is the linear homogeneous Cobb-Douglas one:

$$U = ED_i^{\alpha} Y^{1-\alpha}$$

The specification of the α parameter is a value judgment indicating the weight one wishes to attach to education versus all other goods and services. The higher α, the higher the weight given to education in a household's welfare. The parameter α can be thought of as the proportion of the household's budget that one wishes to see devoted to education. The α parameter could be estimated empirically, on the basis of historical behavior of households. In that case the value judgment would be that households' choice of weights for education are acceptable to society.

The utility level derived from the utility function becomes the argument in the social welfare function:

$$SWF = S(U_i).$$

Again a functional form for the SWF must be specified in order to derive an index of equity. If the SWF used in the previous sections of the paper (D1 bii), is again employed, we have:

$$SWF = \sum_{i=1}^{n} \left(A + B \frac{U_i(ED_i, Y_i)^{1-E}}{1-E}\right)^{\frac{1}{n}}$$

where: $U_i (ED_i, Y_i) = ED_i^{\alpha} Y_i^{1-\alpha}$

n = number of households

The value judgment is contained in the choice of the E (and the α) parameter. The implications of this choice have already been discussed in a previous section.

The final step is to transform the SWF into Atkinson's equity index:

$$I = 1 - \left[\sum_{i=1}^{n} \left(\frac{U_i \ (ED_i, \ Y_i)}{\bar{U}} \right)^{1-E} \frac{1}{n} \right]^{\frac{1}{1-E}}$$

To summarize, an equity criterion resulting in Atkinson's equity index, involves value judgments about the importance of education versus other goods and services (choice of α) and about the importance of households at different levels of utility (choice of E). Having made these choices and having properly measured ED_i and Y_i, one can proceed to calculate Atkinson's cardinal measure, I, and to rank different distributions of ED_i and Y_i.

c. Ability-to-Pay Measure

The literature on tax equity frequently makes use of the concept of ability-to-pay. The concept is most often operationalized by comparing ratios of taxes paid (after shifting) to income, for an average (median) family, across income classes. The ability-to-pay criterion evaluates as more equitable a distribution that is "progressive" or has higher average tax rates as income increases. A related ability-to-pay criterion has been used by Lee Hansen, Burton Weisbrod, and many others, to evaluate equity in higher education.[/] The criterion can be applied to elementary and secondary education as well.

The first step in the construction of an ability-to-pay measure for education is the definition of the burden and/or benefit that will be distributed across incomes. For education equity, a reasonable choice is net benefit, defined as the yearly flow of education

services per household (ED_i) minus the yearly taxes and other fees paid to finance education (TX_i). ED_i would most commonly be measured as dollar flows of education resources to the household. TX_i would be measured as local and state taxes levied to finance the public elementary and secondary school system plus private school costs (tuition). $(ED\text{-}TX)_i$ per household would be the treatment. It represents a change in household resources due to education.

The evaluation of distributional equity of the treatment across households requires, as always, specification of a criterion and a measure. There are at least three possibilities for the ability-to-pay criterion. The first is to employ the regression statistics from the lenear regression of net benefit on income-after-taxes-supporting-elementary and secondary education.

$$(ED\text{-}TX)_i = b_o + b_1 (Income)_i$$

$i = 1, \ldots, n$ where n = number of households

The more equitable distribution would exhibit a negative estimated slope, b_1, because as income increased net benefits would decrease. The problem with this measure is that the coefficient of determination, R^2, is likely to be very low. Households will have varying numbers of children from zero to more than five at each income level and therefore the unexplained or within group variance will be quite high. The source of the problem is that only one service is being measured and when that service is measured on an annual basis, its incidence can vary widely.

One way to avoid the low R^2 problem but remain with the same general criterion, is to divide households into groups based on the number of children in the household. Then using the household

with the median income in each group, net benefits $(ED-TX)_i$ could be calculated. Finally, comparisons of the net benefits and median income levels could be made using linear regression (or simple displays).

$$(ED-TX)_i = b_o + b_1 \text{ (Median Income by number of children)}_i$$

$i = 1, \ldots, j$

j = households by number of children (0,1,2,3...5)

Again, equity would be higher when b_1 was a higher negative number. The within class variance problem would be lessened due to the averaging of benefits for each group, but it is not clear that there would be any relationship in the regression. In order for b_1 to be negative, median family income and net burdens have to be negatively correlated. A negative correlation would be associated with a progressive tax system and/or a negative correlation between numbers of children and median income because these two relationships influence ED_i and TX_i and therefore $(ED-TX)_i$.

A third alternative measure, analogous to the one suggested by Joseph Pechman in a critique of Lee Hansen and Burton Weisbrod's work on higher education, involves grouping households into classes based on income rather than based on numbers of children.[/] Then for each income class the total benefits minus total taxes for all households are divided by the number of households to derive an average net benefit for the class. The magnitude of the net benefit will depend on the structure of the tax system, as before, and on the number of children in the income class. The average net benefit in each class could be regressed on the midpoint of the income class:

$$(\text{Net benefit in income class})_i = b_o + b_1 (\text{Midpoint income class})_i$$

$i = 1, \ldots, k$ $\quad$ k = number of income classes.

Again the more equitable system would have a negative estimated slope, b_1. The R^2 problem would be less likely to occur because the within class variance is now lessened by the use of the average benefits in an income group. What this criterion says is that the unit of analysis is more than the individual household. Now the unit of analysis is the average household in an income class and households with different numbers of children are not distinguished within an income class.

In addition to the problems of choosing which criterion of equity is appropriate, there are several other serious problems with the ability-to-pay specification of equity. First the measurement of education should probably be over a longer period of time than one year. Ideally, because education is in part an investment, a lifetime measure would be desirable. But short of that, a measure that allows for household cycles where children are a part of the household for some years and not others would seem necessary. Otherwise, equity evaluations could change over time depending on the size of the senior graduating class and the entering kindergarten class and how many households switch from being with child to being childless or vice versa.

Second, the measurement of taxes will in practice be difficult whenever education is financed from several revenue sources. At the local level, the property tax usually accounts for education revenues. An education property tax burden can fairly readily be attached to each household that owns a house, but the burden attached

to renters will present a more difficult, but not insurmountable problem. The state financed share of education is usually from the general fund which is supplied by a variety of revenue sources. The measure of tax burden requires that state taxes for education be separated out and distributed to households and this will not be easy.

Finally, the use of income as an independent variable is problematic. An identical problem is present in Atkinson's measure where education services and income are combined in a utility function. Income is supposed to represent the consumption of all other goods and services. Some of those other goods and services will be provided publicly and because they will not all be financed according to the benefit principle, some household's real consumption will be higher (lower) than is indicated by their pre-tax and benefit income. The income measure that is required is one that takes account of the impact of the public sector in all areas except education. Another way to state this problem is to point out that few people would be concerned if net education services were distributed regressively, if all other government activities were progressively distributed. It is the total impact that matters to most people, but the total impact will not be assessed when income is used to represent goods and services other than education.

d. Ex Ante Household Equity

One of the difficulties with the ex ante household equity criterion is the semantic confusion that surrounds the concept.

For that reason this section begins with a series of definitions. Following the definitions, the conceptual basis for the criterion is outlined. Finally a summary of results of the criterion applied to a District Power Equalization System is presented. The derivation of the results is contained in Appendix 3.

i. Definitions

Ex ante equity refers to the determination of the equity of a process (rules and incentives) as outlined on paper and before individuals or school districts have responded to those rules and incentives. Ex post equity on the other hand is concerned with outcomes after complete responses. Ex ante household equity has sometimes been referred to as taxpayer equity or fiscal neutrality. Because households are taxpayers, taxpayer equity is an acceptable alternative label. Fiscal neutrality is, however, a confusing label because sometimes the term refers to a children's concept (lack of correlation between property wealth and expenditures) rather than a household concept. We suspect that some people use the term to refer to both concepts. To avoid confusion, we have eschewed the use of the fiscal neutrality term and have instead more fully described the equity criteria as either non-identical children equity or household (taxpayer) equity.

ii. Conceptual basis for ex ante household equity

Sometimes a District Power Equalization System (DPE) of school finance is equated with ex ante household (or taxpayer) equity. The basis for the equation is that under a DPE equal tax rates will yield equal dollars of spending per child. The criterion of equal rates can be criticized as a taxpayer equity conception for two

easons. First it is not comprehensive enough because it does not include an ability-to-pay standard, such as income, and it does not take into account actual services received or actual taxes paid. Second, even if the idea of equality of the ex ante taxes across households is accepted as a criterion, the tax price not the tax rate is the appropriate focus of concern. The tax price is the rate at which a household can trade private dollars for $1 of spending per child. It is the price of a dollar of spending per child in terms of the household's tax bill. It is the tax bill that a household is concerned about, not the district tax rate. A given property tax rate will translate into different tax prices for households with different house values.

In this section of the paper we address only the second problem with the equation of a DPE and taxpayer equity - the problem of tax prices versus tax rates. We accept as a value judgment the possibility of a desire to define taxpayer equity narrowly as the relationship between tax prices and house values. In such a conception horizontal equity would be defined as equal tax prices for houses of equal value and vertical equity would be defined as higher tax prices for houses of higher value.

iii. Summary of the relationship between tax-prices and house values under a DPE

The conclusions stated in this section are based on the analysis found in Appendix 3. The tax-price of concern is assumed to be the local tax-price or the household's local tax bill per one dollar of spending per child.

Pre-DPE with a non-matching state aid program, neither horizontal nor vertical equity is achieved with respect to tax-prices and house values. Post-DPE equity may exist, depending upon the evaluators' choice to consider education along with other publicly provided goods and services. If education is considered alone, then equally valued houses will pay the same local tax price no matter what their district property wealth is (horizontal equity). If other publicly provided goods and services are considered with education, then tax-prices will be positively correlated with district wealth and horizontal and vertical equity will not exist even with the DPE.

Ex ante household equity is a more narrowly conceived equity criterion than those based on Atkinson's index or on the Hansen-Weisbrod ability-to-pay standard. When the ex ante concept is defined as horizontal and vertical equity of local tax prices with respect to house values, the DPE system meets the standard if education alone is of concern to the evaluators.

III. Hypothetical Numerical Models of School Districts that Illustrate Conflicts in Conceptions of Equity in School Finance

A. The Construction of School District Models

1. The Model as a Static Snapshot

In this paper, the term "school district model" is used to describe hypothetical numerical values for variables important in equity considerations at a point in time in a set of school districts. The sets of school districts are analogous to states. Equity measures are calculated for children or households in each set of school districts and then the sets are ranked from most to least equitable according to the equity measure. Conflicts among conceptions of equity are shown by constructing hypothetical models of sets of school districts such that equity measures lead to conflicting rankings of the sets.

2. The Range of Education and School Finance Variables in School District Models

The complexity of a model of school districts in terms of the number of different variables included in the model is determined by the requirements of the equity conception that will be examined using the model. The models are made as simple as is consistent with the need to demonstrate conflicts in the equity conceptions. In this section, the variables are organized by their use in the models and the models are presented from the simplest to the most complex. Variables that might empirically be important in actual

equity measurement, but are not included in the models, are also discussed. Many times an entire set of variables will have a similar potential effect in the hypothetical models and for that reason only one need be included. The variables may have different effects when actual school district data are employed, but because we are constructing only hypothetical examples and do not know what the data would look like in many cases, it would be repetitive to include those variables whose impact depends solely on their empirical dimensions.

a. Simplest School District Models

Very simple models are all that are needed for the analysis of equity criteria for identical children. The simple models are constructed by specifying the values of two variables for each school district. The two variables are the number of children and the dollar value of expenditures per child.

b. Moderately Complex School District Models

Moderately complex models with more than two variables, are needed for the analysis of equity conceptions for non-identical children. The possible number of variables in addition to the number of children and expenditures per child is large, but we only use two of the possibilities in the actual models. One of the additional variables is used to demonstrate the behavior of equity conceptions for legitimately differentiated children and the other additional variable is used to analyze equity conceptions for non-legitimately differentiated children. The other variables are discussed in this section, but not used in the actual models.

The variables that potentially add to the complexity of school district models can be organized into two categories plus a wealth per child variable. The two categories and the specific variables under each are listed below.

1. Differential Input Price Variables (Personnel Input Prices, Transportation Input Prices, Utility Input Prices)

2. Differential Output Cost Variables
   -- Differential Production Processes
      - Decline or Increase in Enrollments
      - Economies of Scale
      - Transportation

   -- Differential Education Needs (Special Education Programs and Categorical Aid)

   -- Differential Capital Expenditures

The two variables that are used in the actual models are a price index that represents the potential effect of both price and cost variables on univariate non-identical children's measures and a wealth per child variable that represents a non-legitimate children's difference in the bivariate children's measures. Only a price index is needed to represent the potential effect of all the price and cost variables because any (or all) of them can be converted to indexes and used in lieu of or in combination with the price index. Their effect will depend upon their empirical dimensions.[/]

The rest of this section is devoted to a <u>brief</u> discussion of each of the variables that is listed as a differential input price variable or a differential output cost variable. All of the variables can be seen as factors that could change the value of the treatment level per child or factors that could serve to classify children on the basis of legitimate or illegitimate differences.[/ *]

Differential input prices: The price of inputs may differ across children in a state. In such a case, expenditures unadjusted for price differentials will be an inadequate measure of educational resources available to children. The prediction of a possible hypothetical effect of a price adjustment is easier than is the construction of an appropriate price index. Empirical work on price indexes is progressing, but still has not yet produced reliable and valid results.

Differential rates of decline or increase in enrollments and economies of scale: The current number and rate of change of students in a district can influence the quantity and quality of outputs per student. Fixed costs may mean that resources are used inefficiently when enrollments change abruptly and that for a given expenditure level and resource input the level of output varies across districts. For example, school districts experiencing declining pupil enrollment face constraints caused primarily by seniority and tenure provisions of laws or teacher contracts. Even if staff is laid off the remaining staff may be higher than average or desired on both educational preparation and experience thereby resulting in inefficient production processes compared to more stable or growing districts. Economies (and diseconomies) of scale may mean that resource requirements per unit of output vary depending on the district's size, so that once again the same levels of expenditures and resources result in different levels of outputs. The extent to which these factors are important in equity measurement depends both on their empirical magnitude and on the treatment that is chosen (outputs versus inputs).

Transportation: The resource requirements for transporting children to and from school provide one of the most clear-cut examples of cost differentials across districts. For this reason transportation is often considered in a class separate from other, more debatable, cost differences. Transportation costs differ because of population densities, distances from school and perhaps climatic variances. Unless some account is taken of these differentials, dollars of expenditures will translate into differences in real resources across students. Often states provide categorical aid for transportation, but even in such cases there is a need to standardize the resource availability per dollar of revenues if resources are a treatment of concern.

Differential Educational Needs: Children or their treatments are often legitimately differentiated on the basis of the input requirements needed for them to achieve minimum, adequate, or equal outputs. The argument for attention to needs is a vertical equity argument and people vary on the bases on which they choose to establish standards of vertical equity. Some are interested in equalizing some version of final outputs, while others are content if an adequate or minimum output level is achieved for all children taking account of educational needs. When vertical equity is a concern, some adjustment must be made to either treatments or children's weights before the non-identical children can be compared.

Categorical aid presents an especially difficult problem with respect to univariate children's measures. The problem is discussed in more detail in future parts of the paper, but is briefly summarized here. If categorical aid is available to compensate for special education needs (or costs), it is not entirely

clear whether treatments should be indexed (or children weighted) when children's equity measures are calculated. Part of the problem stems from inadequate knowledge about the actual targeting of categoricals in districts: are they spent on special children or are they added to general revenues and spread over all children? Part of the problem stems from the practical difficulty of assigning weights agreeable to all evaluators.

Differential Capital Expenditures: The services from capital equipment and buildings may be a desirable part of a treatment measure if inputs are the treatment. Capital services may vary across children due to the age, quality, and size of the capital stock. The problem is how to appropriately measure capital services. Capital expenditures are not the correct measure because they are lumpy and not an indication of capital consumption rates. On the other hand, their lumpiness may affect a district's ability to consume other resources in the desired proportions.

A second problem with efforts to include capital services in a treatment measure is the different financial arrangements that may exist with respect to debt finance. It is not clear how to account for differences in state aid for debt, differences in the provision of special millages for debt service, and differences in the use of current revenues for debt service or capital expenditures. A common solution is to use current operating expenditures or revenues as a treatment, but this solution may not be appropriate if capital services are important additions to resources or outputs and if they vary significantly across children. Because most of the problems concerning capital require empirical work, we do not use any capital variables in the models.

A third problem in accounting for differences in capital across districts is that school districts experiencing declining enrollments or those located in certain locations may have higher operation and maintenance costs on their capital stock and thus have a need for differential treatment.

c. Complex School District Models

Complex school district models include some or all of the expenditure related variables associated with the previous models plus some combination of the following financial environment variables:

- Ratio of Households with Children to Households Without Children
- Ratio of Children to Taxpayers
- Levels and Distributions of Wealth and Income in Districts
- Divisions of Property Tax Base Between Residential, Commerical, and Industrial Uses
- Combinations of Property and Income Taxation Used to Finance Schools
- Assessment Practices
- Tax Relief and Circuit Breakers
- Municipal Overburden

Complex models are needed for two purposes. Explanations of why and how school district expenditures per child generated require theories of school district response to social, political and financial environments. The financial variables are usually considered crucial in an empirical specification of a theory of school district response. Understanding why and how

outcomes are generated is very important, but it is a vast topic and outside the scope of this paper.

The second purpose of the complex models is to illustrate household (or taxpayer) equity conceptions. If households were the primary concern in this paper, it would be necessary to include many of the financial environment variables in the household models. Because children, not households, are the primary concern, the household models are specified as simply as possible. Some additional complexity generated by the differentiation between residential and non-residential property bases is included in Appendix 3 on tax-prices, but for the most part the household models include ratios of households with and without children, income levels of households, and household tax payments as the only financial environment variables.

B. Introduction to Model Presentations

Table 4 displays the plan for the presentation of school district models. The first column identifies the section in which each model will be discussed. The second column lists the component of the equity conception that will be analyzed with each model, and the third column specifies the variables that will be used in each model. We begin with children's equity conceptions, by addressing identical-children criteria in Section C. In this section the group and the treatment remain constant while the equity criteria are varied to show conflicts among them. Non-identical children's equity criteria are considered in Section D. The conflicts among criteria for legitimate differences among children are illustrated by showing the effect of adding a price index to one of the distributions used for identical children. The conflicts among criteria for illegitimate differences are analyzed by comparing the rankings of a given group, treatment and differentiating characteristic based on alternative bivariate children criteria. The conflicts between households and children as a group are highlighted in Section E, where the criteria and the treatment are held constant and only the group is changed from children to households. Finally Section F demonstrates possible conflicts between different household conceptions. The treatments and the groups are held constant and household equity criteria are varied.

TABLE 4

PLAN FOR MODEL PRESENTATIONS

Section	Component of Equity Conception Emphasized	Variables in Model
C	Identical Children Criteria	Number of Children, Expenditure per Child
D	Non-Identical Children Criteria	
1	- Legitimate Differences	Number of Children, Expenditures per Child, Price Index
2	- Illegitimate Differences	Number of Children, Expenditures per Child, Property Wealth per Child
E	Household and Children Groups	Number of Children, Number of Households, Expenditures per Child, Taxes per Household, Income per Household
F	Household Criteria	Same variables as Section E

FIGURE 1

ANSWERS TO VALUE JUDGMENT QUESTIONS
FOR ELEVEN EQUITY MEASURES
FOR IDENTICAL CHILDREN

	EQUALITY MEASURES										
VALUE JUDGMENTS*	Range	Restricted Range	Federal Range Ratio	Relative Mean Deviation	Permissible Variance	Variance	Coefficient of Variation	Standard Deviation of Logarithms	Gini Coefficient	Percentage of Students Within X% of Mean	Atkinson's Index, E>0
1. All children taken into account?	No	No	No	Yes	No	Yes	Yes	Yes	Yes	No	Yes
2. Improvement for transfers on one side of the mean?	No	No	No	No	No	Yes	Yes	Yes**	Yes	No	Yes
3. Improvement for transfers on one side of the median?	No	No	No	No	No	Yes	Yes	Yes**	Yes	No	Yes
4. Improvement for transfers that cross mean?	No	No	No	Yes	No	Yes	Yes	Yes	Yes	No	Yes
5. Improvement for transfers that cross median?	No	No	No	No	Yes	Yes	Yes	Yes	Yes	No	Yes
6. Sensitive to equal additions?	No	No	Yes	Yes	Yes	No	Yes	Yes	Yes	Yes	Yes
7. Sensitive to equal percentage increase?	Yes	Yes	No	No	No	Yes	No	No	No	No	No
8. Changes at different levels recorded identically?	No	No	No	No	No	Yes	Yes	No	No	No	No
9. Mean for comparison?	No	No	No	Yes	No	Yes	Yes	Yes	No	Yes	Yes
10. Median for comparison?	No	No	No	No	Yes	No	No	No	No	No	No
11. All levels for comparison?	No	No	No	No	No	No	No	No	Yes	No	No

*For a more complete description of the value judgments, see Table 2 and text.
**At very high levels in the distribution, the answer may be no.

to least equitable. Each measure conflicts with at least one other measure in at least one pair of comparisons between the seven distributions.

2. The Models and The Measures

Table 5 displays seven different distributions of expenditures per child. Each distribution has the equivalent of 10 or 11 school districts, with 2100 children distributed among the districts. Each district is represented by a different level of expenditures per child.

Letting distribution A represent the norm for comparison, the other 6 distributions are derived in the following way: distribution E results from a multiplication of each expenditure level in A by 1.1 and Distribution F results from an addition of $150 to each expenditure level. Both E and F increase the mean and median expenditure level from $1500 to $1650. Distributions B, C, D, and G result from mean-preserving changes that reallocate a fixed amount of total expenditures among the children. Distribution B takes $100 per child from the $800 district and gives it to the $500 district and takes $50 per child from the $2350 district and gives it to the $2500 district. Since there are different numbers of children in the giving and receiving districts, expenditure reductions and increases are not always equivalent. The total dollars redistributed does remain constant.

Distribution C reverses the redistribution of B by taking $50 per child from the $800 district and giving it to the $500 district and taking $100 per child from the $2350 district and giving it to the $2500 district. Distribution B results from a larger transfer at the bottom while Distribution C results from a

TABLE 5

NUMBER OF CHILDREN AT EACH PER CHILD EXPENDITURE LEVEL FOR DISTRIBUTIONS A,B,C,D,E,F,G

Distribution A

Expenditures per child	Number of children
$ 500	100
650	3000
800	2000
1000	1000
1200	1000
1500	5000
1800	1000
2000	1000
2200	2000
2350	3000
2500	1000

$: 1500 md$: 1500 total#: 2100

Distribution B

Expenditures per child	Number of children
$ 650	3000
700	3000
1000	1000
1200	1000
1500	5000
1800	1000
2000	1000
2200	2000
2300	3000
2650	1000

M$: 1500 md$: 1500 total#: 2100

Distribution E

Expenditures per child	Number of children
$ 550	1000
715	3000
880	2000
1100	1000
1320	1000
1650	5000
1980	1000
2200	1000
2420	2000
2585	3000
2750	1000

M$: 1650 md$: 1650 total#: 2100

Distribution F

Expenditures per child	Number of children
$ 650	1000
800	3000
950	2000
1150	1000
1350	2000
1650	5000
1950	1000
2150	1000
2350	2000
2500	3000
2650	1000

M$: 1650 md$: 1650 total#: 2100

Distribution C

Expenditure per child	Number of children
$ 600	1000
650	3000
750	2000
1000	1000
1200	1000
1500	5000
1800	1000
2000	1000
2200	2000
2250	3000
2800	1000

$: 1500 md$: 1500 total#: 2100

Distribution D

Expenditures per child	Number of children
$ 500	1000
650	3000
800	2000
1000	1000
1200	1000
1500	5000
1900	2000
2200	2000
2300	3000
2650	1000

M$: 1500 md$: 1500 total#: 2100

Distribution G

Expenditures per child	Number of children
$ 500	1000
650	3000
800	2000
1000	1000
1400	5000
1450	1000
2000	1000
2200	2000
2250	1000
2350	3000
2500	1000

M$: 1500 md$: 1410 total#: 2100

***M$ = Mean expenditure per child

md$ = Median expenditure per child

total# = Total number of children

larger transfer at the top. Both distributions result from transfers on both sides of the mean and median.

Distribution D results from transfers above the mean and median. $100 per child is taken from the $2000 district and given to the $1800 one and $50 is taken from the $2350 district and given to the $2500 one.

Distribution G results from transfers in the vicinity of the mean. $100 per child is taken from the $1500 district. Half the amount is given to the $1200 district and half to the $1800 district. The mean remains at $1500, but the median changes to $1410.

Table 6 displays the value for each of the ten equity measures. The last three measures are all Atkinson's index, with different values for the parameter E. Table 7 shows the ranking of each distribution according to each measure. Table 7 shows that no distribution is ranked the same by all the measures. Put another way, there is at least one conflict between each pair of measures for at least two distributions. An example of at least one conflict is presented for selected pairs of measures in Table 8. The Federal Range Ratio is compared to all other measures and at least one (and usually more than one) conflict is found in all cases. The four measures that are relative inequality adverse (the Gini, Standard deviation of the logarithms, Ceofficient of Variation, and Atkinson's Index) are also compared each to the other.

3. Summary

The numbers used in Models A through G are hypothetical, but within the range of actual school district data. The models demonstrate the possibility of conflicts between measures and the

TABLE 6

COMPUTATION OF EQUITY MEASURES

	Distributions						
	A	B	C	D	E	F	G
1. Range	2000	2000	2200	2150	2200	2000	2000
2. Restricted Range	1700	1650	1600	1650	1870	1700	1700
3. Federal Range Ratio	2.6154	2.5385	2.4625	2.5385	2.6154	2.1250	2.6154
4. Relative Mean Deviation	.3651	.3651	.3651	.3651	.3651	.3319	.3810
5. Permissible Variance	.4792	.4792	.4792	.4792	.4792	.4356	.2878
6. Variance	427401	428116	435259	430020	517156	427401	435735
7. Coefficient of Variation	.43584	.436204	.43983	.43719	.43584	.39622	.44007
8. Standard Deviation of Logarithms	.503	.492	.495	.503	.503	.444	.504
9. Gini Coefficient	.24792	.24656	.24837	.24853	.24792	.22538	.24880
10. Atkinson's Index:							
10a. E = 1.1	.1193	.1161	.1174	.1194	.1193	.0959	.1204
10b. E = 2.5	.2692	.2557	.2590	.2692	.2692	.2169	.2701
10c. E = 8.0	.5170	.4660	.4761	.5170	.5170	.4437	.5170

TABLE 7

RANKING OF DISTRIBUTIONS ACCORDING TO EQUITY MEASURES

Measure	Distribution						
	A	B	C	D	E	F	G
Range	1	1	6	5	6	1	1
Restricted Range	4	2	1	2	7	4	4
Federal Range Ratio	5	3	2	3	5	1	5
Relative Mean Deviation	2	2	2	2	2	1	7
Permissible Variance	1	1	1	1	1	6	7
Variance	1	3	5	4	7	1	5
Coefficient of Variation	2	4	6	5	2	1	7
Standard Deviation of Logarithms	4	2	3	4	4	1	7
Gini Coefficient	3	2	5	6	3	1	7
Atkinson's Index:							
E = 1.1	4	2	3	5	4	1	7
E = 2.5	4	2	3	4	4	1	7
E = 8.0	4	2	3	4	4	1	4

TABLE 8

EXAMPLES OF AT LEAST ONE CONFLICT BETWEEN
SELECTED PAIRS OF CHILDREN'S
EQUITY MEASURES

Pair of Measures	Conflict in Ranking	
Federal Range Ratio and Range	Distributions	A and B etc.
Federal Range Ratio and Restricted Range	Distributions	F and G
Federal Range Ratio and Relative Mean Deviation	Distributions	A and B etc.
Federal Range Ratio and Permissible Variance	Distributions	A and F etc.
Federal Range Ratio and Variance	Distributions	A and E etc.
Federal Range Ratio and Coefficient of Variation	Distributions	A and C etc.
Federal Range Ratio and Standard Deviation of Logarithms	Distributions	A and G etc.
Federal Range Ratio and Gini Coefficient	Distributions	B and D etc.
Federal Range Ratio and Atkinson's Index	Distributions	B and C etc.
Gini Coefficient and Standard Deviation of Logarithms	Distributions	A and C
Gini Coefficient and Coefficient of Variation	Distributions	C and D
Gini Coefficient and Atkinson's Index (any E value)	Distributions	A and C
Standard Deviation of Logarithms and Coefficient of Variations	Distributions	C and D
Standard Deviation of Logarithms and Atkinson's Index (E = 1.1)	Distributions	A and D
Coefficient of Variation and Atkinson's Index (any E value)	Distributions	A and C

consequent possibility of the need to specify value judgments in order to choose one criterion by which to rank sets of school districts. The actual incidence of conflicts when real data are used is an empirical question whose answer is being worked toward by the School Finance Cooperative.[/]

D. Models for Non-Identical Children Criteria

1. Legitimate Differences Among Non-Identical Children and Univariate Measures

a. Introduction

The discussion in Part II, Section D1 c ii, of this paper indicated that legitimate differences among children could be handled by the univariate children's measures in two different ways. One procedure involved the assignment of differential weights to classes of children. The second procedure required the construction of an index that could be used to make treatments for unweighted children equivalent. The two methods are different versions of the same procedure as is demonstrated in footnote _/. In the model in this section we choose the procedure of indexing the treatment so as to make the indexed treatment comparable for unweighted children.

The hypothetical index used in the model is a price index that varies across school districts, but has the same value for all children in a given district. The assumption in the model is that the only legitimate differences among children are those based on differential prices of resources.

b. The model and the measures

The model is constructed by applying a hypothetical price index to the districts in model A from the previous example._/* Table 9 displays the expenditures and number of children in Distribution A, the price index applied to each expenditure level (district), and the price adjusted distribution, labelled AA.

TABLE 9

DISTRIBUTIONS A (NON PRICE ADJUSTED)
AND AA (PRICE ADJUSTED)

Distribution A			Distribution AA	
Expenditures per Child	Number of Children	Price Index	Expenditures per Child*	Number of Children
$ 500	1000	.95	$ 526	1000
650	3000	1.04	625	3000
800	2000	.83	964	2000
1000	1000	1.10	909	1000
1200	1000	.78	1538	1000
[illegible]00	5000	.99	1515	5000
1800	1000	1.17	1538	1000
2000	1000	.89	2247	1000
2200	2000	1.13	1947	2000
2350	3000	.90	2611	3000
2500	1000	1.08	2315	1000
Mean	1500		1532	
Median	1500		1520	
Number of Children	2100		2100	

* = $\frac{\text{Expenditures Per Child A}}{\text{Price Index}}$, rounded to nearest dollar

Table 10 displays the calculated values of the ten equity measures for the price-adjusted distribution AA. If we now assume for simplicity that Distributions B through G in the example in the last section were all previously appropriately price-adjusted, it is possible to show that the change in treatment for A reverses the ranking of that distribution with respect to at least one of the others for each measure. Table 11 shows those conflicts by listing in Column 2 the ranking of Distribution A with respect to a second distribution and in Column 3 the ranking of Distribution AA with respect to the same second distribution. The two columns show a conflict between rankings based on A and those based on AA (price adjusted).

c. Summary

As with the previous models, the conflicts in rankings that result from the price adjustment to Distribution A are only hypothetical. Empirical work is required for an understanding of the actual effects of such adjustments. In a previous paper by Robert Berne[/] some empirical work using data from the State of Missouri was presented. The conclusions based on the Missouri data were that the indexing did not change the ranking of the four distributions of concern, but the size of the equity measures was affected. As discussed in Part II of this paper, ordinal usage of the equity measures is on somewhat firmer ground than is cardinal usage, so that the finding on invariance of the rankings is especially important.

TABLE 10

COMPUTATION OF EQUITY MEASURES

FOR PRICE ADJUSTED DISTRIBUTION, AA

Measure	Value of Measure for AA
. Range	2085.0000
. Restricted Range	1986.0000
. Federal Range Ratio	3.1776
. Relative Mean Deviation	0.3465
. Permissible Variance	0.2974
. Variance	451277.0000
. Coefficient of Variation	.43841
. Standard Deviation of Logarithms	.507
. Gini Coefficient	.24694
. Atkinson's Index	
E = 1.1	.1186
E = 2.5	.2691
E = 8.0	.5180

TABLE 11

CONFLICTS WHEN DISTRIBUTION A IS PRICE ADJUSTED (DISTRIBUTION AA)

Measure	Relationship Between Distribution A and a Second Distribution	Relationship Between Distribution AA and a Second Distribution
1. Range	A equal to B	AA less equal than B
2. Restricted Range	A equal to G	AA less equal than G
3. Federal Range Ratio	A equal to G	AA less equal than G
4. Relative Mean Deviation	A equal to B	AA more equal than B
5. Permissible Variance	A equal to B	AA less equal than B
6. Variance	A more equal than B	AA less equal than B
7. Coefficient of Variation	A more equal than B	AA less equal than B
8. Standard Deviation of Logarithms	A equal to D	AA less equal than D
9. Gini Coefficient	A equal to E	AA more equal than E
10. Atkinson's Index:		
E = 1.1	A equal to E	AA more equal than E
E = 2.5	A equal to D	AA more equal than D
E = 8.0	A equal to D	AA less equal than D

2. Illegitimate Differences Among Non-Identical Children and Bivariate and Multivariate Measures

a. Introduction

In Part II we discussed the use of bivariate and multivariate measures as criteria for the evaluation of illegitimate differences among children. Figure 2 in Part II D biii (reproduced here) summarizes the value judgments inherent in the alternative measures.

In the models of this section, property wealth per child is used as the illegitimate differentiating characteristic among children. The models are constructed with hypothetical data for expenditures per child, number of children at each expenditure level, and property wealth per child. The bivariate and multivariate measures are calculated for each model and a comparison of the measures across models illustrates the differing characteristics of the measures, as summarized in Figure 2.

b. The models and the measures

The models are constructed using two basic bivariate distributions. The first basic bivariate distribution labelled N in Table 12, is constructed as an essentially linear relationship between expenditures per child and property wealth per child, where the expenditures-per-child part of the distribution is identical to distribution A in Table 12 (univariate identical children model). The second basic bivariate distribution is labelled P in Table 12. Distribution P is a basically cubic relationship between expenditures per child and property wealth per child. Distributions N1, N2, N3 and P1, P2, P3 (Table 12) are modifications of the basic distributions N and P. Distributions

FIGURE 2

ANSWERS TO VALUE JUDGMENT QUESTIONS FOR SIX EQUITY MEASURES FOR NON-IDENTICAL CHILDREN

VALUE JUDGMENTS*	Simple Correlation (r)	Simple Regression Slope (b_1)	Simple Regression Elasticity ($b_1 \frac{\bar{W}}{\bar{E}}$)	Constant Elasticity ($\frac{a \ln E}{a \ln w}$)	Quadratic and Higher Order Regression Slope ($b_1 + 2b_2 W^2$)	Quadratic and Higher Order Regression Elasticity ($(b_1 + 2b_2 \bar{W}) \frac{\bar{W}}{\bar{E}}$)
1. Are all children taken into account?	Yes	Yes	Yes	Yes	Yes	Yes
2. Improvement for all mean preserving transfers in dependent variable?	Not Necessarily	Yes	Yes	Not Necessarily	Not Necessarily	Not Necessarily
3. Sensitive to equal additions in dependent variable?	No	No	Yes	Yes	No	Yes
4. Sensitive to equal percentage increases in dependent variable?	No	Yes	No	No	Yes	No
5. Sensitive to equal additions in independent variable?	No	No	Yes	Yes	No	Yes
6. Sensitive to equal percentage increases in independent variable?	No	Yes	No	No	Yes	No

For a more complete description of the value judgments, see Table 3. For exposition purposes it is assumed that E is the treatment (dependent variable) and W is the children's difference (independent variable).

TABLE 12

NUMBER OF CHILDREN, PROPERTY WEALTH PER CHILD,
AND EXPENDITURE LEVELS PER CHILD FOR
DISTRIBUTIONS N, N1, N2, N3
AND P, P1, P2, P3

Distributions N, N1, N2, N3

The same for N, N1, N2, N3		Expenditures per Child			
Number of Children	Property Wealth per Child	N	N1	N2	N3
1000	$10,000	$ 300	$ 550	$ 650	$ 600
3000	15,000	650	715	800	650
2000	21,000	800	880	950	800
1000	25,000	1000	1100	1150	1000
1000	28,000	1200	1320	1350	1200
5000	32,000	1500	1650	1650	1500
1000	35,000	1800	1980	1950	1800
1000	39,000	2000	2200	2150	2000
2000	41,000	2200	2420	2350	2200
3000	59,000	2350	2585	2500	2350
1000	63,000	2500	2750	2650	2400
Total: 2100	Mean: $33,619*	Mean: $1650	Mean: $1650	Mean: $1650	Mean: $1500

*Rounded to nearest dollar

Distributions P, P1, P2, P3

The same for P, P1, P2, P3		Expenditures per Child			
Number of Children	Property Wealth per Child	P	P1	P2	P3
1000	$10,000	$ 500	$ 550	$ 650	$ 600
3000	15,000	650	715	800	650
5000	21,000	1500	1650	1650	1500
3000	25,000	2350	2585	2500	2350
2000	28,000	2200	2420	2350	2200
1000	32,000	2000	2200	2150	2000
1000	35,000	1200	1320	1350	1200
2000	39,000	800	880	950	800
1000	59,000	1800	1980	1950	1800
1000	63,000	2500	2750	2650	2400
Total: 2100	Mean: $28,524*	Mean: $1500	Mean: $1650	Mean: $1650	Mean: $1500

*Rounded to nearest dollar

N1 and P1 multiply each expenditure level of N and P by 1.1 thereby effecting an equal percentage increase in the dependent variable. Distributions N2 and P2 add $150 to each expenditure level in N and P, resulting in an equal absolute increase in the dependent variable. Distributions N3 and P3 make a mean preserving transfer of $100 per child from the highest expenditure level per child to the lowest expenditure level per child.

Table 13 displays the calculated values for the eight bivariate and multivariate measures for each distribution. Except for the cubic slope and elasticity, all the measures show an improvement when distribution N or P is compared to distribution N3 or P3. This is as expected since distributions N3 and P3 lessen the disparity in the dependent variable. The interpretation of the cubic slope and elasticity measures for distributions N and N3 show less equity for N3 than for N. This may be because a cubic functional form is being inappropriately fit to a linear relationship. The interpretation of the negative values for the cubic slope and elasticity measures between P and P3 is ambiguous because it is not clear how to interpret negative values in the first place. On the one hand a high negative value could be interpreted as more equitable than a low one; on the other hand negative values could be interpreted as overcompensation (one might ask why additions to wealth should reduce expenditures per child rather than have no effect on the expenditures).

TABLE

COMPUTATION OF BIVARIATE AND MULTIVARIATE MEASURES

Measure	Distribution							
	N	N1	N2	N3	P	P1	P2	P3
1. Simple correlation	.95776	.95776	.95776	.95307	.35823	.35823	.35823	.33656
2. Simple slope	.04070	.04477	.04070	.03963	.01750	.01925	.01750	.01609
3. Quadratic slope	.04470	.04917	.04470	.04357	.02613	.02874	.02613	.02475
4. Cubic slope	.06723	.07395	.06723	.07011	-.034688	-.038157	-.034688	-.033673
5. Constant elasticity	.98906	.98906	.87243	.94449	.55309	.55309	.48080	.50311
6. Simple elasticity at means	.91220	.91220	.82930	.88822	.33278	.33278	.30252	.27815
7. Quadratic elasticity at means	1.00185	1.00185	.91077	.97652	.49688	.49683	.45171	.42803
8. Cubic elasticity at means	1.50681	1.50674	1.36982	1.57135	-.65962	-.65962	-.59966	-.64032

The comparisons between distributions N, N1, N2 and P, P1 and P2 demonstrate the following results. The elasticities are insensitive to equal percentage changes (N, N1 and P, P1) but sensitive to equal absolute changes (N, N2 and P, P2). The slopes, on the other hand, are sensitive to equal percentage changes (N, N1, and P, P1) but insensitive to equal absolute changes (N, N2 and P, P2). The correlation coefficient is insensitive to either percentage or equal absolute changes (N, N1, N2, and P, P1, P2).

c. Summary

The models in this section have all involved differences in distributions of the dependent variables. They have shown that the correlation, slope, and elasticity measures can conflict in equity rankings between distributions because they respond differently to various kinds of changes in the dependent variable distribution. Although not shown in the models, the measures also respond differently to various kinds of changes in the independent variable. Those responses are summarized in Figure 2.

E. Models for Households Versus Children as a Group

1. Introduction

The choice of a group is such a major decision in the construction of an equity conceptions that it is bound to have an effect on equity evaluations. The models in this section show that effect. The models all apply the same criteria, Atkinson's index, to identical treatments, so that only the group is allowed to vary. The treatments needed in the models are expenditures per child, income per household, and total education tax bill per household. Expenditures per child are applied to the households by assuming each household benefits in proportion to its number of children. If there are no children, the household receives no benefits; if there are two children the household receives two times its district's expenditure level per child, etc. Household income is assumed to be gross total income (before any taxes). The income can represent either annual income or the present value of lifetime income. The latter is an impractical income concept given the current state of measurement capabilities, but in the hypothetical models either income assumption is acceptable. The tax bill is equal to total local, state, and federal unshifted taxes paid for education. In these models, it is assumed that all children attend public schools, so there is no need to include tuition with the tax bill.

2. The Models and the Measures

Table 14 displays the hypothetical data for the 4 models used in this section. The models are labelled R, S, T and V. Each model contains 5 school districts; each school district has either

TABLE 14

HYPOTHETICAL MODELS FOR HOUSEHOLDS AND CHILDREN

MODELS

	R					S					T					V				
School District	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
Number of Households	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	1	1	1	2
Number of Children Per Household	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2 or 1	1	1	1	2 or 1
Expenditures Per Child	$700	800	900	1000	1100	$700	800	900	1000	1100	$800	800	900	1000	1000	$700	800	900	1000	1100
Gross Income Per Household	$10,000	11,000	12,000	14,000	15,000	$10,000	11,000	12,000	14,000	15,000	$10,000	11,000	12,000	14,000	15,000	$10,000	11,000	12,000	14,000	14,000
Total Education Tax Bill Per Household	$500	600	700	1200	1500	$600	700	500	1500	1200	$600	700	700	1300	1300	$700	800	900	1400	1800

1 or 2 households; each household has either 1 or 2 children. Although these models are the most complex ones presented in the paper, they are still relatively simple abstractions of reality.

The differences between the four models are the following: R, S, and T all have 1 household per district and 1 child per household, while V has 2 households per district and 1 or 2 children per household in districts 1 and 5. R and S have identical distributions of expenditures per child and therefore their children's equity ranking will be identical. R and S also have identical income distributions, but their tax bill distributions differ. The difference in tax bill distributions might arise for any number of reasons, including differences in house values (property taxes), differences in consumption and therefore sales-tax payments in cases where the sales tax contributes to the state share of education finance, differences in exemptions or deductions and therefore differences in Federal tax liability, etc. The difference in tax bill distributions means that R and S will differ on the household equity ranking. R and S will show that children as a group can yield identical equity ranking while households as a group yield different equity rankings.

R and T have different distributions of expenditures per child and therefore their children's equity ranking will differ. The income and tax bill distributions in R and T are constructed so that the household equity rankings are identical. R and T demonstrate the same kind of conflict as R and S but now children are different and households the same with respect to equity rankings.

R and V have as identical as possible distributions of expenditures per child. The distributions cannot be perfectly identical because districts 1 and 5 in V have children/household ratios higher than 1.0. The children's equity measures differ because of the differing numbers of children and the household equity measures differ because of the differences in children/household ratios.

The measure or criterion used in all 4 models is Atkinson's Index. The parameter E is set at 2.5. The children's measure necessarily set α at 1.0, because only education expenditures per child and not net income are included as a treatment. (See Section II D2b). The household measures set α at 0.2 and $1-\alpha$ at 0.8; education is weighted 20% and other goods and services 80%.

Table 15 summarizes the calculations of Atkinson's index for each model and each group. As specified above, the following results are achieved. R and S are identical for children but differ for households. R and T are different for children, but identical for households. R and V are different for children and for households.

3. Summary

The group or who is to be the focus of equity concerns is a fundamental choice in equity analysis. The equity evaluation is likely to vary greatly depending on how that choice is made. As always, the models in this section are constructed with hypothetical data. Although the magnitude of the differences shown by the models is an empirical question, there is little doubt that the

TABLE 15

CALCULATIONS OF EQUITY MEASURES

FOR HOUSEHOLDS AND CHILDREN

Model	Atkinson's Index Household Measure	Atkinson's Index Children's Measure
R	.022595	.031388
S	.024400	.031388
T	.022595	.012355
V	.031991	.044822

differences will exist. Because the differences based on the group are likely to occur, a fundamental question about the goals of school finance reform is raised. Many social scientists have pointed out that multiple policy goals require multiple policy instruments. This may also be true in school finance reform, where changing state finance plans may be one instrument incapable of achieving the two goals of children and household equity.

The household models are quite simplistic given the reality of the financial environment in school districts, and for this reason more sophisticated work could be beneficial. The primary focus of this paper is expenditure disparity and children's equity, and for that reason the household models are developed only enough to demonstrate the basic conflict between groups or between household criteria (next section, F).

F. Models for Household Criteria

1. Introduction

The last set of models illustrates the possibility of conflicts between household equity criteria. The group (households) and the treatments (education expenditures, gross income, and tax bill) are held constant while the criterion is varied between Atkinson's Index and the Hansen-Weisbrod slope. All the treatments are defined identically to those in the previous set of models, R through V.

2. Models and Measures

Table 16 displays two models, labelled W and Z. Each is constructed as simply as possible with 6 school districts, 1 household per district and 1 child per household. Model W has identical expenditures per child in all the districts, while in Model Z the expenditures per child differ. The distributions of income and tax bills per household differ between the models. Atkinson's index and the Hansen-Weisbrod slope are calculated for each model. Atkinson's Index sets E equal to 2.5, α equal to 0.2, and $1-\alpha$ equal to 0.8. The calculated values of the equity measures are displayed in Table 17. Model W is more equitable than Model Z according to the Hansen-Weisbrod slope, but less equitable according to Atkinson's Index.

3. Summary

As in the case of children's measures, the measure one uses to evaluate household equity can make a difference in terms of rankings. Atkinson's index is both more sophisticated and more easily related to children's measures, but it is also somewhat more difficult

TABLE 16

HYPOTHETICAL HOUSEHOLD MODELS, W AND Z

	W						Z					
School District	1	2	3	4	5	6	1	2	3	4	5	6
Number of Households	1	1	1	1	1	1	1	1	1	1	1	1
Number of Children Per Household	1	1	1	1	1	1	1	1	1	1	1	1
Expenditures Per Child	$900	900	900	900	900	900	$700	800	900	900	1000	$1100
Gross Income Per Household	$7,000	9,000	11,000	13,000	15,000	17,000	$8,500	9,000	9,500	10,000	10,500	11,000
Total Education Tax Bill Per Household	$850	875	900	900	925	950	$850	875	900	900	925	950

TABLE 17

CALCULATIONS OF EQUITY MEASURES

FOR HOUSEHOLDS

Model	Atkinson's Index	Hansen-Weisbrod Slope
W	.081995	-.009286
Z	.013315	.111429

technically than is the Hansen-Weisbrod slope. In reality it would be difficult to obtain adequate data for either measure, although other researchers have recently managed to do so.[1]

IV. Conclusions About Equity Measurement Based on Conceptions and Models: What Can and Cannot Be Eliminated Based on Theoretical Analyses and Sets of Relatively Acceptable Value Judgments?

A. Introduction

The previous parts of this paper have developed and illustrated many alternative conceptions of equity in school finance systems. Those alternative conceptions have been based on value judgments about one or more of the three components of an equity conception - the group, the treatment, and the criterion. In order to narrow the number of conceptions and thereby reduce the number of possible ways of measuring equity, it is necessary to choose among value judgments for each of the three components. In this section we discuss the kinds of choices that we think are consistent with widely held sets of value judgments and conversely those that we think cannot be made on the basis of relatively acceptable value judgments. The choices that are consistent with widely held values are reasonable candidates for standards by which to narrow the number of alternative measures of equity, but choices that are based on relatively disputed value judgments are ones that cannot be used to reduce the potential number of equity measures. Occasionally we make reference to practical considerations in the choice of equity conceptions, but where this is done it is clearly stated or implied that practical problems are lower order considerations than are the value judgments. By this we mean that in our opinion value judgments need to be explicitly stated rather than indirectly stated as they are sometimes when, for example, practical difficulties such as data

collection, understandability, or manipulation are listed as the only reasons for choices.

This part of the paper is organized as follows: Section B discusses the components of an equity conception that cannot be chosen among on the basis of relatively acceptable value judgments. Choices among the components discussed in Section B must rest on specification of the equity assessor's own values. Section C outlines the components that are able to be eliminated based on widely accepted values. The conclusions from Section B and C combined are summarized in Figure 3 which lists the alternatives in each component of an equity conception. In Figure 3 major categories of alternatives are underlined and options that can be chosen as preferable on the basis of widely held value judgments are capitalized. Figure 3 is more fully discussed in Sections B and C, but one further explanation is appropriate at this point. Every entry in Figure 3, except the one labelled Federal wealth neutrality standard, has been explicitly discussed in previous parts of the paper. The Federal wealth neutrality standard is included here because the OE has specified that it is an alternative to the Federal expenditure disparity measure (FEDM)[/] and a state is allowed to choose between the two the one measure that is most favorable to it.[/*] For completeness we have therefore included the Federal wealth neutrality standard along with the Federal expenditure disparity measure. The Federal wealth neutrailty standard was not included in our previous analyses for two reasons. First, it did not evolve naturally as a major category for either household or children conceptions, and second, our primary efforts have been devoted to expenditure measures because

FIGURE 3

WIDELY AGREED UPON CHOICES IN EQUITY CONCEPTIONS

(Capitalized white entries are preferred above uncapitalized grey entries; uncapitalized white entries cannot be chosen among)
(Major categories are underlined and are horizontally listed; subdivisions of major categories are not underlined and are vertically listed)

Component of Equity Conception	Alternatives for Each Component					
Group	Children - Identical - NON-IDENTICAL			Households		
Treatments	Inputs - Dollar inputs - PRICE ADJUSTED DOLLAR INPUTS - RESOURCES AND QUALITY ADJUSTED RESOURCES	Outputs - Cognitive Skills Measures - Behavioral Output Measures	Impacts - Current - Future	Net Benefits and Income	Tax Price and Wealth	Yield and Effort
Criteria	Univariate - 10 Equity Measures Including FEDM - 10 Equity Measures including FEDM but excluding range, restricted range, and variance - GINI COEFFICIENT, STANDARD DEVIATION OF LOGS, COEFFICIENT OF VARIATION, ATKINSON'S INDEX	Bivariate or Multivariate - Slopes - Correlation Coefficients - Federal Wealth Neutrality Standard - ELASTICITIES		Atkinson & Hansen-Weisbrod		Federal Wealth Neturality Standard

they are the main concern of this report.

The Federal wealth neutrality measures are included in two places in Figure 3 as an indication of the confusion over whether it is fundamentally a children's measure or a household measure. It can be interpreted as an ex ante bivariate children's measure because it can be argued that equal yields for equal effort, the principle upon which the Federal wealth neutrality standard is based, is one interpretation of the construction of a system in which child's education does not depend on the wealth of his/her neighbors. On the other hand, the Federal wealth neutrality standard could be seen as a household measure if one thinks that equal yield for equal effort is meant to make the tax system for education more equitable across taxpayers. Further work is needed to determine if there are yet other household conceptions that may have been overlooked in this paper.

B. Components That Cannot Be Chosen Among on the Basis of Widely Held Value Judgments

1. The Group

The choice between children and households is not widely agreed upon. The lack of consensus on the appropriate group is mirrored in the Office of Education's dual equity standards; the expenditure disparity standard clearly uses children as the group while the fiscal neutrality standard could be interpreted as a household or taxpayer standard.[/] Academic literature is also divided in its choice of a group. Usually an implicit choice of one or the other group is made, but the chosen group is not consistently either children or households.[/*] Occassionally, academic researchers have argued against a household measure that is based on the net benefits of only one government program, on the grounds that it is the distributional effect of all government activity that is of concern.[/**] This argument does not seem to have convinced all researchers and thus we conclude that it is an insufficient basis on which to choose children rather than households as a group. In addition, in our analysis we have not been able to identify a conceptualization that can combine the two groups into one measure, making the choice between the groups or their use simultaneously unavoidable.

2. The Treatments

Children's treatments must be chosen among the broad categories of inputs, outputs, and impacts. Except on the basis of availability of valid and reliable data and direct control over the treatment, there is no widely agreed upon choice among the three. Inputs are

most often used as the treatment both because of the data availability and because inputs are directly under the control of the school system, but many people conceptually prefer outputs or impacts.

Household treatments can be divided into two choices. An ex post treatment is composed of net benefits (current or future education services minus tax burden) in relationship to current or future income. An ex ante treatment is composed of the tax price in relationship to property wealth. There does not seem to be a widely held preference for one versus the other, although economists seem to choose the ex post one more often and school finance analysts seem to choose the ex ante one more often.

3. The Criteria

Among the broad classes of choice for both children and households there are no widely acceptable preferences. For the children's group, both univariate and multivariate measures are used as criteria, sometimes in combination with each other. In part the lack of choice between univariate and bivariate measures is due to differences in the concept measured.

For the household group and the net benefit-income treatment, both Atkinson's index and Hansen-Weisbrod measures are used by researchers. Thus far practitioners have not tried to adopt either measure, so our conclusion of a lack of concensus on which measure to use is based on academician's research reports.

4. Conclusions

Figure 3 divides each of the three components of an equity conception into major categories and then subdivides each major

category. The major categories are all underlined and listed horizontally, while subdivisions are not underlined and are listed vertically. On the basis of this section's discussion, we can conclude that none of the major categories can be eliminated on the basis of relatively agreed upon value judgments. In Section C we will see that only at the subdivision level can some choices be made.

C. Components That Can Be Chosen Among on the Basis of Widely Held Value Judgments

Figure 3 and the discussion in Section 2 show that no components of a household equity conception can be eliminated. Therefore the entire discussion in this section will refer to subdivisions of the major categories of choice in a children's conception (see the left-hand columns of Figure 3).

1. Children's Group

Children as a group can either be considered as identical or non-identical. There is widespread agreement that the non-identical group is preferred. In addition there is agreement on some of the classifications that serve to differentiate the non-identical children and further there is some agreement on which of the classifications should be viewed as legitimate and which as non-legitimate. There is no agreement on how to quantify the legitimate differentiating classifications. For example, there are many differences across states in the weights given to handicapped versus non-handicapped children, even though there is almost universal agreement that handicapped children should be more heavily weighted. The problem of quantification of agreed upon qualitative distinctions is extremely serious from the point of view of a quantitative equity measure. The Office of Education's regulations give states wide discretion. We will discuss this procedure in more detail in Parts IV and V.

In this section we are content to list some of the qualitative distinctions among non-identical children that are based upon widely accepted views. Clearly agreed upon legitimate differences are most

of the ones that the Office of Education has called "Special Cost Differentials". Quoting from the Office of Education regulations, these differentials are:

> "(i) Those associated with pupils having special educational needs, such as handicapped children, economically disadvantaged children, non-English speaking children, and gifted and talented children; those associated with sparsity or density of population, cost of living, or special socioeconomic characteristics within the area served by an agency."/

In addition to legitimate differentials there are a number of non-legitimate differentials that are widely agreed upon. These non-legitimate classifications include property wealth of the district and race, ethnicity, and sex of the children. Although conceptually the legitimate, non-legitimate distinction is clear-cut, empirically, problems may develop if a legitimate and a non-legitimate differential are correleated. For example, ethnicity and bilingualism are likely to be correlated so that measures of equity may have trouble differentiating them.

2. Children's Treatment

Although choices among the major categories of inputs, outputs, and impacts cannot be made, within the input category some subdivisions are preferred over others. Price adjusted dollar inputs and/or resources and quality adjusted resources are widely agreed to be more desirable conceptually as treatments than are dollar inputs. Problems with the preferred treatments involve the construction of valid and reliable empirical measures of them. Although this is a serious problem when equity measures must be quantified, it is similar to the problem of quantifying legitimate differences among children. A solution must be found, but the data and construction problems

do not obviate the relatively clear-cut conceptual preferences.

3. Children's Criteria

Univariate versus bivariate criteria cannot be chosen without significant disagreement, but within each category some relatively widely held value judgments can narrow the choices.

a. Univariate Criteria

In previous parts of the paper, we have identified ten univariate equity measures.[/] Figure 1 (Part IID) listed the value judgments consistent with each one. One value judgment that is fairly widely held is that the measure should display constant relative inequality aversion.[/*] Only three of the measures, the range, the restricted range, and the variance, are not relative inequality adverse. If these three measures are excluded, then there are seven fairly widely agreed upon ones to use as univariate criteria.

Two additional value judgments that are fairly widely held are that all children should be taken into account and that transfers from children higher in the distribution to those lower in the distribution should be taken into account most of the time. If these two judgments are combined with constant relative inequality aversion, only four measures remain appropriate for use as univariate equity criteria. These four measures are the gini coefficient, the standard deviation of logarithms, the coefficient of variation, and Atkinson's index.[/**] Although the narrowing of the measures to the four just named is based on three value judgments, the judgments are probably widely enough agreed upon to warrant such a narrowing.

b. Bivariate or Multivariate Criteria

The choice among multivariate measures can be narrowed by once again adopting the constant relative inequality aversion value judgment. The judgment is particularly appropriate for the independent variable where its value can be defined as arbitrary percentages of a standard, such as property wealth as a percent of full market value. That value judgment for either the dependent or independent variables (or both) eliminates the slope measures, leaving the correlation coefficient and the elasticities. If, in addition, the judgment that the measure should be sensitive to equal additions in the dependent variable (treatment) is made, then only the elasticities remain as appropriate measures for multivariate criteria. The correlation can also be eliminated because it is not necessarily sensitive to mean preserving transfers, while at least one elasticity is sensitive. There is probably fairly widespread agreement on the equal percentage value judgment on either the equal addition or mean preserving transfer one as well. Therefore we conclude that slope measures and the correlation coefficient can be eliminated, and that the elasticities are the preferred measure.

V. Value Judgments Inherent In The Regulations That Define And Describe The Federal Expenditure Disparity Measure

The Office of Education in its role as executive for Congressional laws has necessarily made a number of very specific decisions about equity conceptions in order to arrive at one quantifiable measure to apply across all states. Those decisions on the definitions of the group, the treatment, and the criterion are briefly summarized here. Following the summary, the alternatives not chosen by the Office of Education are discussed. During the discussion value judgments involved in Office of Education choices are highlighted.

It should be emphasized at the start of this part that the discussion of Office of Education decisions and choices in this report is in the context of the conceptual framework described in Parts I, II and III of the report. We are not evaluating the extent to which the Office of Education did or did not follow Congressional intent as stated by the language of section 5d(2) of P.L. 81-874 as amended by P.L. 93-380. Therefore some of the decisions we describe may be ones that were necessitated by the language of the law and alternative decisions might require a change in that law. Our purpose in this report is to evaluate the FEDM in a broad context.

A. Decisions on the Group

In its specification of the Federal expenditure disparity measure, the Office of Education (hereafter OE) uses a children's

group concept rather than either a district or a household concept. The district is an alternative employed by many school finance scholars in their evaluations of the equity of individual state equalization programs. In an individual state evaluation, especially one conducted for a state legislative body, the district may be an appropriate unit of analysis if the impacts on the political jurisdictions relevant to each legislator are to be highlighted. At the Federal level, the intra-state political considerations are not as relevant and the children's concept is more reasonable. The choice between the children and the household concept is a value judgment and the lack of consensus on the preferred group is possibly reflected in the OE's specification of a Federal wealth neutrality standard to be used at the discretion of each state.

Further specifications of the children's group made by the OE include the exclusion of all non-public school children and the specifications of the definitions of the pupil count and of the appropriate ways to combine data on pupils in states with a variety of school district types. The OE's decision on pupil count has been to allow each state to use the pupil count it employs in its state aid program and if the state uses a pupil weighting or other system to account for student differentials, to allow the state to choose whether or not to employ the system or selected parts of it. The state is to choose between pupil counts corrected for differentials and those not corrected on the basis of the one that yields the most favorable value of the equity measures.

The OE's decision on different district types is again to allow each state to choose between two alternatives, the one most

favorable. One alternative is to ignore different district types and the second alternative to calculate a weighted average of the equity measure calculated separately for each district type and weighted by the proportion of children in each type. The OE assumes that the second alternative will be adopted in most cases.

By allowing the states to make their own choices the OE eschews the decision-making role. At the same time it eschews the possibility of making cross state comparisons on a consistently defined standard. In the rest of this section we briefly discuss the different possibilities for choices of pupil counts and combinations of varying district organizations with the objective of identifying the kinds of decisions that would have to be made if a consistently defined standard were to be computed for all states.[/]

1. Pupil Count Alternatives

Pupil count measures differ depending on the basic population count and depending on how pupils within the basic population are weighted. The basic population count can be a membership-based measure or an attendance-based measure. The membership measure includes either all public school pupils that a district must support financially if they choose to enroll (membership) or all public school pupils that a school district must provide with services if they choose to enroll (enrollment).[/*] The attendance-based measure is calculated from the number of pupils in attendance on a certain day(s) during the school year. Attendance counts will never exceed membership counts and will almost always be smaller. The identification of the gainers and losers that result from a choice of one measure over the other depends upon the relationships between the

alternative measures and districts or pupil characteristics such as urbanness, percent of disadvantaged pupils, etc. For example, larger urban school districts usually have a high absentee rate. Thus, the pupil counts in these urban districts will be higher compared to other districts if a membership rather than an attendance based figure is employed.

Given a decision about membership-based versus attendance-based counts, further choices must be made about relative weights for different categories of pupils. The weights can be constructed as simple multiples of 1.0 or they can be translated into classroom or instructional units. In the latter cases, standards are set for numbers of pupils (and for teachers) per classroom or instructional unit with the possibility that the standards can vary across different categories of pupils. The value judgments involved in weighting systems depend both on the categories of pupils chosen for special weights (qualitative decisions) and on the actual value of the numerical weights assigned to the categories (quantitative decision). An alternative to the current OE procedure of allowing wide discretion to states on how they weight pupils is to develop a uniform weighting system to be applied in all states for calculation of the Federal equity measure. This alternative may or may not be consistent with Congressional intent, but is is not as far-fetched an alternative as it might at first seem to be because of the increasing involvement of the Federal government in the provision of funds and/or the issuance of regulations for two large special groups of pupils -- Title I disadvantaged and handicapped. Currently there are data on a nationwide basis for Title I eligible pupils and

soon there will also be data on handicapped pupils. These data could be utilized, along with Congressional wording on both programs and weights from states now using such weights to establish uniform weights for these two groups. If this were done then all states could be required to include the categoricals directed to these pupils in the numerator and the weights for the pupils in the denominator when calculating the Federal measure. There would not, of course, be any need to specify that the uniform weights be used in the state program or that categoricals necessarily be established for the two groups by a state. The only requirement would be a consistency in the calculation of the Federal measure.

2. Alternatives for Combining Measures for Varying District Organization Types

There are at least three alternative ways to calculate a single value for an equity measure when a state is composed of varying district types. An example of varying district types is the division of a state into K-12 (unified), K-6, and 7-12 districts. The OE choice of separately calculated measures that are combined with weights equal to the proportion of pupils in each district type is one alternative. A second alternative is to ignore district types and calculate one measure based on the expenditures per pupil regardless of district origin. A third alternative is to construct fictional unified district (K-12) for all districts by pairing all non-unified districts until K-12 combinations are achieved.

The OE alternative of a weighted average of the separately calculated measures is not entirely satisfactory because the weighted average is unlikely to yield the same value of the equity

measures as would result if all districts were unified. At the extreme if each district were made into a unique type, the weighted measure would be zero. Of course, the measure based on unified districts only would not be zero. In general an analogy can be drawn between the variance of a sum of two variables and the weighted average OE measure. Just as it is incorrect, except in the case of independent random variables, to equate the sum of the variance of two variables to the variance of the sum, thereby ignoring a positive or negative covariance, so it may be incorrect to sum the equity measures for multiple district types thereby ignoring the "between group" interaction. The OE measure for multiple district types may be lower or higher than it would be for one district type, but it is unlikely to be the same.

The second alternative, to ignore differing district types, is problematic if non-unified districts have different average levels of expenditures based upon some appropriate differentiating characteristics (such as grade level) of the pupils. In such a case the unified districts will average out the difference but the non-unified districts will not. The third alternative of fictional combinations will be difficult to achieve in practice due to non-overlapping district boundaries.

3. Summary

The OE choices can be labelled "pluralistic" in that they allow diversity depending on state choices. The problem with such a pluralistic approach is that each state may be evaluated on a unique standard, making cross-state comparisons difficult. The problem cannot at present be entirely resolved both for practical

data reasons and because of a conceptual lack of development with respect to weights. Some recommendations on how to progress somewhat further in terms of comparability across states are discussed in Part IV.

B. Decisions on the Treatment

This section will follow the same outline as the previous one on the group decisions. First, the OE choices will be summarized and then alternatives to those choices will be briefly enumerated.

The OE has made four specific choices that affect the measurement of the treatment. First, the treatment can be specified as either expenditures or revenues. Second, the expenditures or revenues must be current and thus cannot include outlays for capital or for debt service. Third, Federal funds accountable to the Federal government (such as Title I funds) cannot be included. Fourth, revenues or expenditures taken into consideration under the State aid program that are designated for "cost differentials" can be included or omitted, wholly or partially, at the discretion of the state. There are a number of alternatives to each of these choices and those alternatives are discussed next.

1. Revenues or Expenditures

When the dollar measure is close to total dollars there is little difference between revenues and expenditures. If, however, subtractions from the total are to be made there will be differences in what can be eliminated depending on whether the revenue or the expenditure measure is used. Revenues can usually be examined by source, including local, state, and federal sources. These individual sources can be divided further by type of program such as general and categorical for state sources or Title I and impact aid for Federal sources. Expenditures can be identified by purpose such

as operating, debt service and capital expenditures. The operating expenditures category can be further divided into instruction, utilities and maintenance, transportation, food service, etc. And instructional expenditures can be further subdivided. The OE dollar measure is fairly comprehensive, but in some states both revenue and expenditure data will be needed for calculation and for that reason it is incorrect to say the two are alternatives. For example, revenue data are needed to eliminate Federal funds accountable to the Federal government, and expenditure data may be needed to eliminate debt service if debt service revenues are combined with operating revenues. As another example, if there is both a state and local contribution to transportation and the local contribution is made from the general fund, then in order to eliminate transportation from the OE measure, revenue data would be required for the state contribution while expenditure data would be required for the local contribution. Although OE could consistently require either an expenditure or a revenue measure, there would be no purpose unless a much narrower dollar measure (such as instructional or classroom teacher dollars) were to be used and in some cases the specification of one or the other would make impossible the presently specified inclusions and exclusions.

2. Current Dollars

The obvious alternative to the use of current dollars is to include dollars spent on capital as well. Current dollars already include expenditures on maintenance of the capital, but all other

outlays associated with capital are excluded. If it were possible to assign a dollar value to depreciation and to capital's rate of return (opportunity cost), then a real alternative to current dollars would exist. But since this is beyond the state of the art, the choices must be made among available information on caiptal outlays and debt service. Between the two, debt service is preferable to capital outlays because it is not as lumpy and better approximates the value of the stream of services over time yielded by the capital. However, debt service is also an imperfect measure because its yearly time sequence is sometimes determined by factors other than the life of the investment purchased and its rate of utilization. It is possible for example, for one school district to be using the services of a building for which it has no debt service, while another district makes large debt service payments on a building that will exist long after the entire debt is repaid. The use of debt service to represent current capital utilization is clearly not very accurate. Conceptually, there is a case for including the use of capital services in a total current dollar measure but the available measures are very weak.

3. Accountable Federal Funds and Cost Differentials

Many of the revenues or expenditures identified as accountable Federal funds and cost differentials are also known as categoricals because they are available for certain limited categories of expenditures. The OE regulations allow states wide discretion on whether to include or exclude these funds, except that accountable Federal funds must be excluded. This discretion is fairly clearly intended

to be consistent with the law, Section 5d(2) of P.L. 81-874 as amended by P.L. 93-380. This section reads in part:

> "The terms 'State aid' and 'equalize expenditures' as used in this subsection shall be defined by the Commissioner by regulation after consultation with State and local educational agencies affected provided that, the term 'equalize expenditures' shall not be construed in any manner adverse to a program of State aid for free public education which provides for taking into consideration the additional cost of providing free public education for particular groups or categories of pupils in meeting the special educational needs of such children as handicapped children, economically disadvantaged, those who need bilingual education, and gifted and talented children."

The purpose of this report is not to determine if the OE has followed Congressional intent nor to determine if the OE could have chosen alternative ways to interpret Congressional intent. The report's purpose is, rather, to evaluate the FEDM in the context of a conceptual framework, as developed in the first three parts of the report. Therefore the comments we make, anyplace in parts IV and V or the report and the ideas we describe for possible modifications of the FEDM and its use, may either be capable of implementation by the OE or they may require that Congress change the language of Section 5d(2). Again we do not purport to have studied the extent to which the OE followed Congressional intent and therefore none of our comments should be construed to reflect on this issue.

A more consistent treatment of categorical type funds across states could be effected if all such funds were either included or excluded or if the same ones were excluded (included) for all states. The consistency is a serious one for two reasons. First, states have constructed state finance systems with differing shares of total dollars in categorical (general) classifications and sometimes the

reasoning behind the shares has been an equalization one. If equalization motives play a (large) role in the construction of categorical as well as general aid programs, then there is a good case for making sure that both categoricals and general aid are considered together in an equity measure. Second, the problem of the inconsistency of the usage of pupil weighting systems is heightened by the inconsistency in the treatment of categoricals. Categoricals and pupil weights often refer to the same classifications of students and if a state chooses to use one but not the other, difficult-to-interpret equity measures can result.[/]

The exclusion of accountable Federal funds will be problematic if states have built their own finance systems in response to the Federal funds. In most cases, such as Title I, it is illegal for a state to give less state aid than it would have without the Federal program, so even if a state has managed an illegal substitution of Federal aid in place of state aid, it is probably still appropriate to exclude Federal funds from the calculation of the FEDM. The problem arises if a state has set up a weighting system and its own state aid categoricals in such a way that the weighted pupils are treated 'equitably' from the state's point of view only if Federal funds are included. The regulations allow flexibility in the pupil count used in the calculation of the equity measure, but as discussed previously, although possibly consistent with the law, this causes inconsistencies across states and between numerators and denominators of the expenditure per pupil measures. To some extent, then, the problems of how to treat state "cost differentials", pupil weights, and accountable Federal funds are interdependent.

An additional problem arising from what OE has broadly labelled "cost differentials" is that price levels for educational resources may vary within a state. The OE could require that a price index be developed for each state and the Federal measure be calculated on the basis of price adjusted current dollars. The relaistic possibility of such an alternative depends on how quickly the technology of price indices can be perfected. It is important to note that the problem is one of intra not inter state price differentials. Interstate price differentials will not affect the equity measures if a relative inequality averse measure is used because the between state price level differences are equivalent to equal percentage increases for all treatments in a state.

C. Decisions on the Criterion

The Federal expenditure disparity measure defined by the OE is the same as the measure we have labelled the Federal range ratio. The value judgments involved in its use are listed in Figure 1 and summarized here.

Value Judgments Implicit in Use of Federal Expenditure Disparity Measure

1. Some of the pupils are not taken into account.
2. Improvement is not always shown for mean preserving transfers.
3. There is decreasing absolute inequality aversion (sensitivity to equal additions)
4. There is constant relative inequality aversion (no sensitivity to equal percentage increases)
5. Changes at different levels of the distribution are recorded differently.
6. Neither the mean, the median nor all levels is used as a basis of comparison.

Based on the discussion in Part IV on preferable characteris of criteria, we may note that the Federal expenditure disparity measure exhibits two of the less preferable characteristics. Fi it does not include all the pupils in the measure. The justifica tion for the exclusion is that the tails of a distribution should not be overly influencial in a measure. There are, however, othe ways to circumvent the problem of atypical tails, namely by the use of measures that give less weight to the values in the tails. In addition, many people consider the lower tail, at least, very important. Second, the Federal expenditure disparity measure fai

to show improvements for transfers unless the 5th and/or 95th percentile expenditures are involved. This is a somewhat peculiar judgment because most people would attach some importance to pupils in the middle of the distribution and probably in at least the lower tail as well.

The OE has a number of alternative equity criteria from which to choose. These alternatives are all the univariate measures previously discussed (see Figure 1). The alternatives may be narrowed to those with the three preferable characteristics of constant relative inequality aversion, inclusion of all pupils, and sensitivity to most mean preserving transfers. There are four measures that satisfy these characteristics, those being the coefficient of variation, the gini coefficient, the standard deviation of logarithms, and Atkinson's index.

The final decision for the criterion is the way in which equitable and inequitable situations are determined. This decision is really composed of two separable issues. Fisrt, there is the issue of which states are sufficiently equitable so that they may include impact aid as local revenues, second, for states that can include some portion of impact aid as local revenue, there is the question of what portion.

The existing regulations treat these two issues separately. Only states that exceed the 25% cutoff for the FEDM can count impact aid as local revenues. However, the portion of the impact aid that can be so counted is determined by a different criterion that is described in the provision of the law that specifies the use of equalized local revenues as a percent of total revenues.

Note that although the existing regulations include the treatment of the two issues separately this need not be the case. In theory it would be possible to use either the FEDM or the equalized local revenue criterion as the single criterion for any state and district within the state provided that the portion of impact aid counted as local revenue does not exceed the provision in the law regarding equalized local revenues.

Note also that the current utilization of the FEDM is as a cardinal cutoff. Although it is preferable to use the univariate equity measures ordinally (for ranking) rather than cardinally, the OE has either choice if it wishes to set an absolute rather than a relative (i.e. top 10% of the states) standard. There are, however, choices involved in the application of the absolute standard. The OE standard is applied to make equity a black and white phenomenon; a state's school finance system is either equitable ($\leq$ 25% disparity) or inequitable (> 25% disparity). An alternative to the cutoff is a more gradual standard where degrees of equity are recognized. A further discussion of a graduated equity standard will be presented in Part VI (Recommendations). In this section the graduated standard is cited as an alternative to the current regulations.

D. Conclusions About The OE Decisions On Equity Conception

Many of the OE's specifications for calculation of an equity measure are the only ones that are reasonable in the light of data availability or conceptual development. For example, in this category are the allowance of either expenditures for capital. Other specifications are not so clear-cut, given the alternatives available. Three aspects of the regulations in particular stand out as in need of further thinking. The three are the criterion, the application of the criterion in a non-graduated way, and the wide latitude given the states to include or exclude cost differentials and pupil weights. The last two are especially important with respect to their intended and unintended incentive effects. Wide latitudes on cost differentials and weights could encourage states to manipulate their state finance systems in order to score better on the Federal measure if there were significant amounts of money available. The non-graduated criterion could discourage progress in states most in need of it -- those furthest from the equity cutoff, although it must be recognized that the encouragement of equity may not be a goal of the program. Many of the aspects of the regulations discussed in this part will be referenced in the next one when the recommendations are presented.

VI. Recommendations

In this part of the paper we discuss the recommendations that follow from the previous analyses. Throughout the paper many issues relating to equity measurement have been discussed. However, for the most part, the recommendations focus on the use of disparity measures in general and the FEDM in particular since these measures are the overriding goal of this report. Before we discuss these specific recommendations, a brief comment on the selection of the conceptions of equity to include in Federal regulations is presented.

Conceptions of education equity can be formulated by selecting the group, treatment, and criterion that comprise the conception. One of the major conclusions of this paper is that there are many conceptions of equity that can be formulated and that each incorporates numerous value judgments. A summary of the various combinations of group, treatment, and criterion was presented in Figure 3 in Part V. Our recommendation regarding the various conceptions of equity is that OE should explicitly identify the decision process by which certain conceptions are included in Federal regulations and ot conceptions are excluded. Since the selection of a conception is a choice among value judgments, we are recommending that OE identify t procedure by which they select among these value judgments.

Once the possibility is recognized that multiple conceptions ma be called for, even after applying standards expressed by Congress c the courts, OE will be faced with the problem of utilizing two or mo standards at the "same time". With this possibility in mind two additional recommendations are put forward.

First, since there is likely to be conflict among the conceptions of equity, OE should use multiple measures and should explicitly decide whether to use them alternatively (satisfy one or the others), simultaneously (satisfy all at once) or in parallel (satisfy one of three for 1/3 credit, two of three for 2/3 credit, etc.). The option to use the measures alternatively is more pluralistic and the least demanding since it allows greater discretion to the states and does not centralize the decision-making authority. The simultaneous option, while the most stringent, might be appropriate if the conflicting conceptions are all strongly held goals. The parallel option is a compromise between alternating and simultaneous. Complete equity on all conceptions is not required for "credit" as in the simultaneous case, and equity on one conception does not equate to credit for complete equity as in the alternating case. As the law is currently written, the consideration of these three options will interact with the proportionality requirement for equalized local revenues.

Second, there is a need to undertake continuing empirical work in order to measure the probable extent of actual rather than hypothetical conflicts resulting from choices.[/] The school district models of Part III highlighted many potential conflicts, but the data were imaginary and in fact constructed to yield disagreements rather than agreements. Actual school district behavior may yield substantially less conflict than is present in the hypothetical models. Although empirical results would be helpful, we register one caveat concerning their potential use. Data on past behavior

can never be used with 100% certainty to predict or define the future and this is most true when changes are rapid and non-marginal and theories of change are primitive. In the world of school finance there have been and continue to be significant changes, the effects of which are not always accurately predicted by theories. For that reason empirical results should not be the overriding determinant of equity conception choices, but rather value judgment differences should be kept in the forefront. Otherwise we may find that two conceptions that empirically do not yield conflicts this year may begin to do so in the future and a choice based on empirical agreement may turn inadvertently into a value judgment choice.

Regardless of the procedures utilized by OE or others to select equity conceptions, based on our reserach and analysis it appears likely that a disparity measure that focuses on children will be utilized in Federal regulations. With this in mind, the recommendations regarding PPEDM, specifically the FEDM, are presented in four sections. First, a series of recommendations regarding the group, treatment, and criteria in the FEDM are put forward. The second set of recommendations is related to the issue of interstate comparability. Third, the use of a specific value of the criteria as a cutoff is discussed and alternatives presented. Finally, a set of recommendations is put forward that suggests additional ways to make choices among alternative equity conceptions.

A. Recommended Group, Treatment and Criteria in Disparity Measure

1. Group

Disparity measures are generally selected when concern focuses on children. Since this is the case, children are the obvious group for any PPEDM including the FEDM. Not only are children the preferred conceptual perspective, but also a disparity measure should be calculated using the children (or pupil as it is sometimes called) unit of analysis. This procedure has the effect of weighting each district in a state's distribution by the number of pupils in the district when a disparity measure is calculated. Since children are the focus, this weighting gives each child, rather than each district, equal importance and is therefore recommended. Note that the FEDM as currently defined in the OE instructions is calculated using the children or pupil unit of analysis and we recommend that if alternative or additional disparity measures are utilized, then they too should be calculated in this way.

2. Treatment

Our analysis has concluded that there is no clear-cut conceptual agreement about the choice among the three major treatment categories of inputs, outputs, and impacts. There is, however, widespread agreement that the measurement of outputs is difficult and the measurement of impacts almost impossible given our present state of knowledge and skill. Measurement problems are generally not insurmountable and for that reason we recommend continued support of efforts to work toward satisfactory measures of outputs and impacts.

In the meantime, the input treatment should be as close as possible to the dollar value of real resources. This means that expenditures and revenues should ideally be price and cost adjusted. As we have previously mentioned, price and cost indices are not yet reliable, valid and available for all states. A number of researchers are continuing their efforts to refine such indices and these efforts should be closely followed so that their methodologies can be adopted for the Federal measure in the future. Until further research is undertaken, the use of simplified price or cost adjustments is not recommended since it is not clear that some adjustment is more accurate than none at all.

3. Criteria

Our analysis has not identified a best expenditure disparity measure, but it has eliminated several members of the per pupil expenditure disparity measure class as less desirable than others Four per pupil expenditure disparity measures stand out as most preferable on the basis of three value judgments. Those four measures are the coefficient of variation, the gini coefficient, the standard deviation of logarithms and Atkinson's Index. We recommend that the OE consider the use of one or more of these measures in place of the current Federal expenditure disparity measure. Two of the measures, the gini coefficient and the coefficient of variation, are fairly widely used by educational analysts and would not be extraordinarily difficult to introduce. Atkinson's Index and the standard deviation of logarithms are less well-known, and for that reason possibly more difficult to introduce. Furthermore, the standard deviation of logarithms does not show an improve

ment for mean-preserving transfers at the very high end of the distribution and may be unsatisfactory for that reason. However, Atkinson's Index might be worth the extra explanatory effort because of its flexibility in setting exact value judgments and because it might be used in [illegible]sehold measure as well.

B. The Issue of Comparability Of Treatment And Pupil Counts Across States

In any Federal regulation of state programs there is a tension between the need to take legitimate state differences into account and the desire to design regulations that have meaning. In their 1974 report, "Public Law 874 and State Equalization Plans: The Problems of the Legislative Prohibition of Section 5(d)(2)", the Library of Congress recognizes this tension in the setting of equalization criteria (see page 30). This tension is also present when the decisions on how to take into account student needs and State categoricals are considered.

We find that there are conceptual problems with the method by which the current regulations are structured with regard to student needs, pupil counts, and state categoricals. The law does offer specific guidelines on this issue and we are insure whether our recommendations are within the bounds of Congressional intent. Nevertheless, the issue is important so that we put forward these recommendations recognizing that it may imply a change in the regulations and/or the law.

Ideally the <u>treatment</u> would include all local and state (and perhaps Federal) revenues or expenditures (including all catigoricals) and pupils would be counted and weighted on a unifrom basis across states. This ideal is not presently possible because the quantitative and to some extent the qualitative dimensions of weighting systems are not widely agreed upon. As an intermediate step it would be preferable to have each state be consistent in its own choices such that <u>all</u> categoricals are either included or

excluded and student weights, if available, are used when categoricals are included but not used when they are excluded. Furthermore, if weighted pupil measures are included in the calculation of the disparity measure, they should be an integral, not a peripheral, part of the state aid program. This procedure would begin to move toward an interpretable measure where the treatment and the pupil count were consistent with each other, if not across states. The procedure would, at the same time, eliminate some of the discretion for manipulation which possibly serves as an incentive to construct school finance programs so that at least one combination of treatment and pupils will score high on an equity measure. While it may seem farfetched, under the current instructions a state could construct a weighted student measure for a very small categorical program and then utilize this peripheral weighting scheme in the calculation of the disparity measure. The recommended change could be easily implemented by simply removing the total discretion the states now have over both treatment and pupil count utilization.

In order to increase interstate comparability and utilize as broad a measure of resources as possible by including categoricals or special cost differentials in the treatment, we recommend that OE should seriously investigate the possibility of using a Federally defined weighted pupil count in the calculation of the resource/pupil variable and the disparity measure. Since data on disadvantaged pupils and handicapped pupils are or will be available for all districts, conceptual development of and agree-

ment on weights, not data availability, are the problems to overcome to implement this recommendation.

C. The Problems Associated With The Current Regulations As Incentives To Increased Equalization

The current regulations are designed to identify states that have an equalization program while, at the same time, protect Federally impacted districts. The current regulations are not designed as incentives to encourage states to develop more equitable school finance plans. However, since there is tendency in any organization to utilize existing procedures rather than search for alternatives, the incentive effects of the current regulations should be indicated in case the current regulations are suggested for use as incentives in the future. This is especially important given the current focus of the Federal government on equalization in the states.

There are both theoretical and incentive effect reasons why consideration should be given to alternatives to a strict cutoff level dividing equitable states from inequitable ones. Theoretically we are on firmer ground when an equity measure is used ordinally rather than cardinally.[/] The OE has little choice but to use a measure cardinally because reankings that specify only an order are not satisfactory for setting standards toward which states can strive. That is, an ordinal usage of the measure would mean the regulations would have to say that the bottom X% of the states were inequitable and no matter how equitable every state program were to become, there would always be a bottom X%. Given the need for cardinal rather than the preferable ordinal usage, a graduated standard would be less strict and somewhat less removed from ordinality.

The incentive effects of a strict cutoff are inappropriate in that the most inequitable states may be the most discouraged rather then the most encouraged to improve their finance systems. Also, once having achieved the admittedly arbitrary Federal standard a state has no incentive to improve further. The current regulations imply that there are no decreasing "costs" of inequity as the equity cutoff is approached, but rather that a little inequity (26%) is as "costly" as a lot (126%). Just as the costs or benefits of most other phenomena (such as pollution on the bad side and a well-nourished population on the good side) are accumulated gradually rather than suddenly, it is possible that the costs of inequity are gradually increased as a state's school finance system achieves a higher and higher value of the Federal expenditure disparity measure. Thus, if incentives to equity are a goal states should be rewarded for this gradual improvement.

In order to circumvent the problems involved in using a strict cutoff for the Federal measure, we recommend that a graduated scale for equity be considered if incentives are a goal. The graduated scale could coordinate values of the Federal measure with percentages of Federal impact funds to be counted in states' programs. The selection of the values of the measure and the percentages would necessarily be somewhat arbitrary, but could perhaps be guided by empirical data on the current levels of equity measures across states so that the low equity and high equtiy levels provide states an obtainable and continuous reward. For example, the curre 25% cutoff is obtained by about four states and is therefore quite a difficult standard. For that reason it may be decided that a state is 95% equitable if the 25% cutoff is met but that, for examp

90% equitable if a 30% level is met, 85% if a 35% level is met, etc. The subsequent levels and percentages below 25% and 95% could be determined by a review of the actual levels of the states at present. It is always difficult to set an incentive scale, even when the concept for the incentive, such as the external social cost of pollution levels, if known. With equity incentives, we have no efficiency standard such as external costs to use, making the problem of scaling more difficult yet. However, just as pollution standards can be set using empirical data and knowledge of incentives, so could the equity standard be set on a "practical" basis, yielding possible improvements both theoretically and incentive-wise.

D. Further Refine Choice of Equity Measures through Concurrent Use of Value Judgment Choices, Empirical Analysis, and Practical Standards of Desirability

Perhaps the primary finding of this analysis has been the extent to which the choice of a measurable equity standard depends on a selection among different value judgments. There is no way that we have discovered to circumvent the necessity for value judgments, but there are three procedures that may be combined with the value judgment to help rationalize choices. First, the OE could encourage discussion of the value judgments among interested school finance groups in order to ascertain if there is significant agreement on any of the values. For example, views might be solicited from Congress, Congressional staffs, state department of education personnel, judicial proceedings, policy analysts and researchers. Although statistical equity measures are not likely to generate avid interest even among school finance groups, it is possible that a wider airing of the issues could reveal a wider consensus than has been so far ascertained in our analysis.

Second, further empirical work in addition to that already undertaken by NCES, the School Finance Cooperative,[1] and other analysts, could be helpful in determining which hypothetical conflicts among conceptions are likely to appear in the actual data. Although conflicts that do not exist in present data cannot be entirely ignored because they may appear in future data, the currently existing conflicts could provide short-run guidance on areas of most concern.

Finally, after agreement on value judgments and empirical work have narrowed the range of conflicts as much as possible,

practical standards of desirability might be devised to give guidance to the remaining choices. The practical standards could include the following:

- Cost of Data Collection and Calculation

Preferable measures of treatments, pupils or criteria will require varying amounts of resources to be allocated to data collection and criterion calculation. A judgment about the benefits of further refinement versus the costs could be (and no doubt already is) used by the OE. Attention should be paid to the possibility that initial investments of resources to learn calculation methods or accumulate better data may yield lower future costs as well as increased benefits in equity measurement. Specifically, if the OE were to switch from the use of the Federal expenditure disparity measure to one of the more preferable univariate measures, an initial increase in calculation time spent by state departments of education or OE personnel might result. However, the one time increase in learning time would quickly diminish and that investment in time might be warranted by the benefits of a preferable measure.

- Understandability of Measure to Concerned Public

It is important that users of the Federal equity measure be able to interpret its meaning when it is applied to different state school finance systems. Among the important users are Congressmen and women and their staffs, state department of education personnel, school teachers, and educational analysts. Some of our recommendations, such as comparability of treatments and pupil counts should improve understandability. Other recommendations, such as the selection of a different summary measure, might temporarily

decrease understandability within some groups. The decrease would be expected to be temporary and perhaps quickly overcome if a somewhat familiar measure such as the coefficient of variation or the gini coefficient were chosen.

- Error Proneness of Quantification of Conception

It is desirable that whatever conception is chosen as a standard for quantification be capable of accurate calculation so that users can be reasonably sure that replication can occur without significant changes in the results. Sometimes conceptual improvement may have to be traded against error proneness, especially in the early stages of the development of a new procedure such as a price or cost index. In such cases the OE might opt for low levels of error proneness until new procedures can be standardized.

In summary, while the choice of an equity measure cannot be made devoid of value judgments, it is possible that wider discussion of values, further empirical analysis, and practical standards of desirability can be used to make choices less arbitrary and as consistent with widespread consensus as possible.

FOOTNOTES

PART II.

Page 10

/Additional arguments in favor of children as a unit of analysis are persuasively presented by Stephen Michelson, "What is a 'Just' System for Financing Schools? An Evaluation of Alternative Reforms," in "Future Directions for School Finance Reform," editor Betsy Levin, Law and Contemporary Problems, School of Law, Duke University, Winter-Spring 1974.

/*Peter Stowe has made the following suggestion on how to think about private school children:

> "An alternative criterion might be that the subgroup excluded from the equity criterion may be so excluded if that subgroup has taken action, to exclude themselves from being directly affected by the agent concerned with equity. That is, society is concerned with educational equity of children and chooses as a vehicle to deliver educational equity the financing of public schools. Those children (households) who willfully remove themselves from the vehicle of control take upon themselves the burden of obtaining an equitable education and hence need not be objects of concern to the State."

Page 11

/A similar taxonomy is presented in Jack Leppert, Larry Huxel, Walter Garms, and Heber Fuller, "Pupil Weighting Programs in School Finance Reform," School Finance Reform: A Legislators' Handbook, edited by John J. Callahan and William H. Wilken, National Conference of State Legislatures, Washington, D.C. 20036, February, 1976.

Page 15

/Of course, a district with higher costs may also have higher prices.

Page 20

/A value judgment dispute over the unit of analysis is illustrated in an exchange between W. Lee Hansen and Burton Weisbrod on one side and Joseph W. McGuire on the other in "Communications: The Distribution of Subsidies to Students in California Public Higher Education: Reply and Rejoinder," Journal of Human Resources, VXIII, No. 1, Winter 1978. J. McGuire claims that the relevant group for discussions of equity in California's system of higher education is families with heads aged 55-60. W. L. Hansen and B. Weisbrod respond on page 138: 'The question of which population should be used as a base for comparison with the population of families of college students has no 'correct' answer ... In any case, it is not clear that McGuire's comparison is 'more useful' than the one we made. Moreover, we see no basis in scholarly research for terming our comparison 'improper', 'irrelevant', biased' ..., or 'invidious' ..., we simply adopted a different approach than McGuire prefers."

Page 21

_/For example, we can describe the prevailing weather conditions of a particular location using criteria such as temperature, wind conditions, precipitation, etc. However, the location's weather advantages or disadvantages depend upon one's values. Airline pilots, farmers, umbrella manufacturers, taxi drivers all are likely to have different values.

Page 24

_/For similar treatments see H. M. Levin, "The Effect of Differen Levels of Expenditure on Educational Outputs" in R. L. Johns et al eds The Economic Factors Affecting the Financing of Education, (Gainesville Florida: National Education Finance Project, 1970); H. M. Levin, "Effects of Expenditure Increases on Educational Resource Allocation and Effectiveness," in J. Pincus, School Finance in Transition, Cambridge, Mass: Ballinger, 1974); J. E. McDermott and S. P. Klein, "The Cost-Quality Debate in School Finance Litigation: Do Dollars Make a Difference?" in B. Levin, ed. Future Directions for School Finance Reform, (Lexington, Mass.: Lexington, 1974); S. Michelson, "What is a 'Just' System for Financing Schools? An Evaluation of Alternative Reforms," in B. Levin, Future Directions; R. Berne, "Equity and Public Education: Conceptual Issues or Measurement" Working Paper No. 4, Public Policy Research Institute, Graduate School of Public Administration, New York University, October, 1977.

_/*See also W. N. Grubb and S. Michelson, States and Schools, (Lexington, Mass.: Lexington, 1974).

Page 26

_/See W. I. Garms, J. W. Guthrie, L. C. Pierce, School Finance: The Economics and Politics of Public Education, (Englewood Cliffs, N.J. Prentice-Hall, 1978).

Page 27

_/See F. N. Kerlinger, Foundations of Behavioral Research, 2nd edition, (New York: Holt Rinehart, Winston, 1973), pp. 443 ff.

_/* Ibid.

_/**Ibid.

_/***This is similar to the notion of interpersonal comparisons in utility functions. See Appendix 1 and A. Sen, On Economic Equality (NY: W. W. Norton, 1973).

_/****See Kerlinger, op. cit. pp. 437-438.

_/*****See Appendix 1. Difference here is defined up to a positive linear transformation.

Page 29

_/See, for example, E. E. Lawler III, Motivation in Work Organization, (Monterey, Cal: Brooks/Cole, 1973), especially Chapter 3 on expectancy theory.

Page 33

_/Revenue and expenditure measures can be combined. An example would be local and state revenues less expenditures for debt service.

Page 34

_/See Berne, op. cit., pp. 49ff, where data from Missouri, unadjusted and price adjusted, are examined.

Page 35

_/J. S. Coleman et al, Equality of Educational Opportunity, (Washington, D. C.: U.S. Government Printing Office, 1966).

_/*See J. S. Akin and I. Garfinkel, "School Expenditures and the Economic Returns to Schooling," Journal of Human Resources, 12, Fall, 1977, pp. 460-481; G. E. Johnson and F. P. Stafford, "Social Returns to Quantity and Quality of Schooling" Journal of Human Resources, 8, Spring, 1973, pp. 139-155; C. R. Link and E. C. Ratledge, "Social Returns to Quantity and Quality of Education: A Further Statement," Journal of Human Resources, 10, Winter, 1975, pp. 78-89. For some contrary evidence see R. D. Morgenstern, "Direct and Indirect Effects on Earnings of Schooling on Socio-Economic Background," Review of Economics and Statistics, 55, May, 1973, pp. 225-233; and T. I. Kibich and J. L. Murphy, "The Economic Returns to Increased Educational Spending," Journal of Human Resources, 10, Winter, 1975, pp. 56-77.

Page 36

_/Reference here is to the National Center for Education Statistics' Elementary-Secondary General Information Survey (ELSEGIS).

Page 37

_/Price adjustments are also considered as differentiating characteristics among different children in an earlier part of the paper.

Page 37 (continued)

_/*See H. Brazer, "Adjusting for Differences Among School Districts in the Costs of Educational Inputs: A Feasibility Report," in E. O. Tron, ed., Selected Papers in School Finance 1974,(Washington, D.C.: Office of Education). H. Brazer and A.P. Anderson, "A Cost Adjustment Index for Michigan School Districts," in E. O. Tron, ed., Selected Papers in School Finance, 1975,(Washington, D.C.: Office of Education,1975); J. G. Chambers, A. Odden and P.E. Vincent, Cost of Education Indices Among School Districts,(Denver: Education Commission of the States, December 1976);W. N. Grubb and J. Hyman, "Constructing Teacher Cost Indices: Methodological Explorations With California Unified School Districts," in E. O. Tron, ed., Selected Papers in School Finance, 1975; L. W. Kenny, D. Denslow and I. J. Goffman, "Measuring Differences Among the Florida School Districts in the Cost of Education: An Alternative Approach," in E. O. Tron, ed., Selected Papers in School Finance, 1975;and E. O. Tron, "Introduction and Summary," in Selected Papers in School Finance, 1975.

Page 39

_/Akin and Garfinkel indirectly test this possibility.

Page 40

_/Florida, for example, builds cost adjustments in the state aid distribution system. See W. N. Grubb, "Cost of Education Indices: Issues and Methods" in J. J. Callahan and W. H. Wilken, eds. School Finance Reform: A Legislators' Handbook, (Washington, D.C.: National Conference of State Legislators, 1976).

Page 41

_/An example would be "1000 books equals one teacher."

Page 42

_/G. E. Pugh, I. N. Killalea, and B. Loatman, Educational Opportunity, the Concept, Its Measurement and Application,(Washington, D.C.: DHEW, NCES, 1978). The measure discussed here is labelled M4 in Pugh et al, see pp. 31ff.

Page 43

_/See S. J. Carroll, T. S. Donaldson, H. J. Kiesling and J. Pincus, How Effective Is Schooling? A Critical Review of Research,(Englewood Cliffs, N.J.: Education Technology Publications, 1974) and E. A. Hanushek, "A Reader's Guide to Educational Production Functions" paper presented at NIE National Invitational Conference of School Organization and Effects, 1978 for reviews.

Page 43 (continued)

_/*See S. J. Klees, "Instructional Technology and Its Relationship to Quality and Equality in Education in a Developing Nation: A Case Study of Instruction Television in Mexico", unpublished Ph.D. dissertation, Stanford University, 1975; R. Murnane, The Impact of School Resources on the Learning of Inner City Children,(Cambridge, Mass.: Ballinger, 1975), and A. S. Summers and B. L. Wolfe, "Do Schools Make a Difference?" American Economic Review, 67, September, 1977, pp. 639-652.

Page 45

_/See, for example, P. Taubman and T. Wales, Higher Education and Earnings,(NY: McGraw-Hill, 1974),pp. 175-177; S. Bowles and H. Gintis, Schooling in Capitalist America,(NY: Basic Books, 1976) and H. M. Levin, "Effects of Expenditure Increases on Educational Resource Allocation and Effectiveness," for discussions of cognitive skills.

Page 47

_/Kerlinger, op. cit.,p. 493.

_/*See Hanushek, op. cit., p. 10ff.

_/**See Using Achievement Test Scores to Allocate Title I Funds, A Report to the National Institute of Education,(Washington, D.C.: DHEW, NIE, September, 1977).

Page 48

_/Ibid.

_/*See R. Berne, "Education and Earnings: Expectations and Outcomes. The Experience of Recent High School Graduates," unpublished Ph.D. dissertation, Cornell University, 1977, for a review of earnings functions.

_/**See especially Bowles and Gintis, Schooling in Capitalist America, W. H. Sewell and R. M. Hauser, Education, Occupation and Earnings,(NY: Academic Press, 1975),and Taubman and Wales, op. cit.

Page 49

_/See Using Achievement Test Scores to Allocate Title I Funds, op. cit., p. 12ff.

_/*In Using Achievement Test Scores to Allocate Title I Funds estimations of the cost to produce a national data base for achievement scores are presented and the interested reader is urged to consult this document. Depending upon a set of assumptions, three year testing costs are estimated to range between $7 and $55 million.

Page 52

_/Also, many of the earnings functions are only estimated for high school graduates.

Page 54

_/See Appendix 1

_/*R. Inman, "Optimal Fiscal Reform of Metropolitan Schools," American Economic Review, 68, March, 1978, pp. 107-122.

Page 55

_/Inman's formulation also allows for private school spending and tax capitalization effects but these are not considered here.

Page 61

_/Much of the work on the value-judgment approach is taken from an earlier paper by Robert Berne,"Equity and Public Education: Conceptual Issues of Measurement," Working Paper No. 4, Public Policy Research Institute, New York University, October 1977.

_/*Appendix 1 provides more detail on technical aspects of SWF's and their possible use in the evaluation of equity in education.

_/**All of these characteristics, as well as some others, are more fully described in Appendix 1.

Page 62

_/Anthony Atkinson, "On the Measurement of Inequality," Journal of Economic Theory, V. 2, 1970.

Page 63

_/Ibid., p. 250.

Page 65

_/Amartya Sen, On Economic Inequality, (NY: W. W. Norton, 1973); Amartya Sen, Collective Choice and Social Welfare, (San Francisco: Holden-Day, 1971); D. M. G. Newberry, "A Theorem on the Measurement of Inequality," Journal of Economic Theory, Vol. 2, 1970; P. Dasgupta David Starrett, Amartya Sen, "Notes on the Measurement of Inequality," Journal of Economic Theory, 1972.

_/*The SWF used in Atkinson's Index is also symmetric.

_/**The SWF used in Atkinson's Index is not concave when E = ∞.

Page 66

_/Appendix 1 more fully describes the difference between ordinal and cardinal.

Page 67

_/R. Berne, op. cit.

Page 70

_/The human imagination is very creative when it comes to inventing new measures, so it cannot be said that the list is an all-inclusive one.

_/*Appendix 2 to this report contains a definition and, where appropriate, the mathematical formula for each measure.

_/**There are many other range ratios that could be considered, but they all exhibit the same value judgments as the federal range ratio. Some of these other measures are:

1. The highest district current operating expenditure (COE) per pupil divided by the lowest district COE/pupil.
2. The ratio of COE/pupil for each decile to the COE/pupil for the 95th percentile.
3. The ratio of COE/pupil for each quartile to the COE/pupil for the 95th percentile.
4. The ratio of COE/pupil for each decile to the norm or mean COE/pupil.
5. The ratio of COE/pupil for each quartile to the mean COE/pupil.
6. The ratio of the interquartile difference in COE/pupil to the mean COE/pupil.

Page 75

_/See the papers by Harvey E. Brazer and Ann P. Anderson, "A Cost Adjustment Index for Michigan School Districts" and W. Norton Grubb and James Hyman, "Constructing Teacher Cost Indices: Methodological Explanations with California Unified School Districts," in Selected Papers in School Finance, 1976, USOE, USDHEW, for alternative views about which causes are allowed to count.

Page 76

/An alternative way to represent the kind of information provided by a linear regression slope is to present the mean or median value of the dependent variable for each decile of the independent variable. If there is a correlation between the dependent and independent variables, then it should show up as an increase (decrease) in the mean (median) of the dependent by decile of the independent. This kind of presentation is attractive because it is easily comprehended by statistically unsophisticated readers.

In addition to these measures, Alan Hickrod has devised a way to construct a "bivariate Gini coefficient," often cited as Hickrod's Gini. If expenditures per pupil is the dependent variable and property value per pupil is the independent variable, the Hickrod Gini would be constructed as follows. Expenditures per pupil would be ordered on the basis of the corresponding property value per pupil, where property value per pupil is arranged from low to high. Then the percent of property value would be placed on the X axis and the corresponding percent of expenditures on the Y axis. The first point would be (0,0) and the last (1,1) as usual. But in between the Hickrod-Lorenz curve could cross the 45° line. In other words the Y value (percent of expenditures) could exceed its corresponding X value (percent of wealth). In such a case, the Gini coefficient calculation is not clear.

/*Appendix 2 contains the formula for each of these regression statistics.

Page 85

/The discussion in the paper has benefited greatly from Robert Inman's work, op. cit., and "Micro-Fiscal Planning in the Regional Economy, A General Equilibrium Approach," Journal of Public Economics, April 1977, pp. 237-60.

/*This assumption is problematic because some of the other goods and services are publicly provided and taxes paid and services received may not be equal. Robert Inman (op. cit., 1978, p. 129) has specified Y to equal total annual income minus local and state school taxes minus private school tuition plus the annual value of the capitalized changes in the value of a household's residential plot.

Page 87

/W. L. Hansen and B. A. Weisbrod, "The Distribution of Costs and Direct Benefits of Public Higher Education: The Case of California," Journal of Human Resources, Spring 1969, pp. 176-191; J. A. Pechman, "The Distributional Effects of Public Higher Education in California," JHR, Summer 1970, pp. 361-370; R. W. Hartman, A Comment on the Pechman-Hansen-Weisbrod Controversy," JHR, Fall 1970, pp. 519-523; J. W. McGuire, "The Distribution of Rubsidy to Students in California Higher Education," JHR, Summer 1976, pp. 343-353.

202

Page 89

/J. Pechman, op. cit.

PART III.

Page 97

_/This paper is a conceptual one and although there is some existing research on the empirical dimensions of many of the listed variables, a review of such research is outside the scope of this paper.

_/*Many of the variables are discussed in other parts of the paper.

Page 98

_/See references to price index work in Part II, Section C.

Page 100

_/See R. Berne and L. Stiefel, "A Methodological Assessment of Educational Equality and Wealth Neutrality Measures," A Report to the School Finance Cooperative, 1978, for a further discussion and some empirical results on alternative ways to account for capital services.

Page 105

_/We do not use the percentage of student measure in the models because it is neither commonly utilized in practice nor theoretically superior to other measures.

Page 113

_/See Robert Berne and Leanna Stiefel, "A Methodological Assessment of Educational Equality and Wealth Neutrality Measures," in "A Report to the School Finance Cooperative," July, 1978. The Report contains calculations on the nine measures and others for over 30 states for several years in the 1970's.

Page 114

_/The equivalence of indexing and weighting is dependent on the correct specification of the index. We will use the following symbols to show the equivalence in a simple and a more complex example.

T_{i1} = treatment level in district i, children of type 1 (for example, treatment level for handicapped).
T_{i2} = treatment level in district i, children of type 2 (for example, non-handicapped).
P_{i1} = number of children in district i, type 1 classification (handicapped).
P_{i2} = number of children in district i, type 2 classification (non-handicapped).
$P_i = P_{i1} + P_{i2}$.

Page 114 (continued)

$$T_i = \frac{P_{i1} T_{i1} + P_{i2} T_{i2}}{P_{i1} + P_{i2}}$$

= unadjusted average treatment per child district i.

W_i = weight assigned to all children in district i (for example, price differential weight).
W_1 = weight assigned to children type 1, all districts.
W_2 = weight assigned to children, type 2, all districts.

Example one: Adjustment made for price differences only across districts.

1. Weighting Procedure:

$$\text{Weighted } T_i = \frac{P_{i1} T_{i1} + P_{i2} T_{i2}}{W_i (P_{i1} + P_{i2})}$$

2. Indexing Procedure:

$$\text{Indexed } T_i = \frac{T_i}{W_i} = \frac{\left(\frac{P_{i1} T_{i1} + P_{i2} T_{i2}}{P_{i1} + P_{i2}} \right)}{W_i}$$

$$= \frac{P_{i1} T_{i1} + P_{i2} T_{i2}}{W_i (P_{i1} + P_{i2})} = \text{weighted } T_i$$

Example two: Adjustment made for price differentials and handicapped, non-handicapped.

1. Weighting procedure

$$\text{Weighted } T_i = \frac{P_{i1} T_{i1} + P_{i2} T_{i2}}{W_i (W_1 P_{i1} + W_2 P_{i2})}$$

2. Indexing Procedure

$$\text{Indexed } T_i = \frac{P_{i1} T_{i1} + P_{i2} T_{i2}}{\left(\frac{W_i (W_i P_{i1} + W_2 P_{i2})}{P_{i1} + P_{i2}} \right)}$$

$$P_{i1} + P_{i2}$$

$$= \frac{P_{i1} T_{i1} + P_{i2} T_{i2}}{W_i (W_1 P_{i1} + W_2 P_{i2})} = \text{Weighted } T_i$$

where index $= W_i (W_1 P_{i1} + W_2 P_{i2})$

Page 114 (continued)

_/*Although the index is hypothetical, the range of values is chosen from empirical work by Harvey Brazer and Ann P. Anderson on cost indices. See H. Brazer and A. P. Anderson, "A Cost Adjustment Index for Michigan School Districts," in E. O. Tron, ed., Selected Papers in School Finance, 1975 (Washington, DC: Office of Education, 1975).

Page 116

_/Robert Berne, op. cit., 1977, pp. 52ff.

Page 135

_/See Robert Inman, "Optimal Reform of Metropolitan Schools", op. cit., and Susan C. Nelson, "The Equity of Subsidies for Higher Education", Papers in Education Finance, Paper No. 5, Education Commission of the States, Denver, Colorado, February 1978.

Page 137

_/We have previously called the measure the Federal Range Ratio.

_/*The Federal wealth neutrality standard is a complicated measure to calculate, but the principle upon which it is based is the simple one of equal yield for equal tax rates. Section 115.64 of the Federal Regulations of March 22, 1977 gives examples of how to calculate the Federal wealth neutrality standard.

Page 140

_/The fiscal neutrality standard could also be interpreted as a children's concept on the grounds that a "non-neutral" financing system discriminates against the children in districts with low property values per child.

_/*For an example of the choice of the household as the group see Robert P. Inman, "Optimal Fiscal Reform of Metropolitan Schools", The American Economic Review, March 1978. For an example of children see Anita A. Summers and Barbara L. Wolfe, "Intradistrict Distribution of School Inputs to the Disadvantaged: Evidence for the Courts", Journal of Human Resources, Summer 1976, pp. 328-42.

_/**For an example of this agrument, see Joseph A. Pechman, "The Distributional Effects of Public Higher Education in California", Journal of Human Resources, Summer 1970, pp. 361-70.

Page 144

/"Instructions for Determining the Percent of Expenditure or Revenue Disparity in a State School Finance Program Under S. 115.63 of Federal Regulations of March 22, 1977", p. 2, U.S. Office of Education.

Page 145

/The ten measures are the range, the restricted range, the Federal range ratio, the permissible variance, the relative mean deviation, the variance, the coefficient of variation, the standard deviation of the logarithms, the Gini coefficient, and Atkinson's Index.

/*This judgment is consistent with a desire to control for inflationary increases if dollar values are not in real terms and if inflation is uniform across school districts.

/**It should be noted that the standard deviation of logarithms does not show an improvement for all transfers in the upper part of the distribution, while the other three show improvements for transfer anywhere in the distribution.

Page 149

/We intend to keep the discussion brief because the choices we outline have been discussed by many others and are not new. It is important, however, to reference these chioces in preparation for the recommendations in Section VI.

/*Differences in membership and enrollment will depend on the balance between the number of pupils exported to other districts versus the number imported from other districts.

Page 158

/Further discussion of this issue of consistency between the numerator (treatment) and denominator (pupil count) will be found in Part VI.

Page 165

/A recent thorough exploration of using data for over 20 states is available in "A Methodological Assessment of Education Equality and Wealth Neutrality Measures, A Report to the School Finance Cooperative", by Robert Berne with the assistance of Leanna Stiefel, July 19

Page 168

_/For example, see the analysis in Pugh, et al, op. cit.

Page 173

As discussed in Appendix 1, it is possible that even an ordinal measure goes further than the average person's ability to make equity decisions. Amartya Sen has suggested that incomplete rankings are closer to most people's intuitive assessments of equity.

Page 176

_/National Center for Educational Statistics, 1978, op. cit., and Berne, Stiefel, July 1978, op. cit.

APPENDIX 1

Technical Appendix on Applications of Social Welfare and Social Choice Analysis to Measurement of Equity in Education

INTRODUCTION

The evaluation of equity in social systems has been approached in two different, and until recently, unrelated ways. One approach has wrestled with the highly abstract and theoretical problems of how the quality and quantity of society's well-being associated with alternative social states can be determined. This literature has developed concepts of Social Welfare Functions (SWF) and Social Choice Functions (SCF) to aid thinking about how to make that are essentially redistribution decisions. The Social Welfare Function, which was developed first, is conceived of as a way to assign a real value (number) to every possible configuration of states of the world. The Social Choice Function is both more and less demanding. It is more demanding because it requires specification of the way in which members of a community or group join to make value judgements on the goodness and badness of alternative social configurations (i.e. how the Social Choice Function is generated) It is less demanding because it need not result in a real valued function that assigns a number to every alternative. In fact, it may be no more than a ranking of better, worse and non-comparable.

This literature on social decision-making has remained separate, until the early 1970's, from the work of empirical

researchers who statistically measure the degree of economic inequality. The empirical researchers have been using measures such as gini coefficients and coefficients of variation for both positive and normative purposes. The normative purposes have been pursued by either ranking different distributions (incomes across countries, earnings by sex etc.) from most equal to least equal or by using the empirical measures cardinally to say how much more equal one distribution is than the other.

Because the empirical measures can be used normatively to either rank or assign a real value to alternative distributions, they have a clear parallel to SWF'S or SCF'S. A number of scholars such as Amartya Sen, Anthony Atkinson, and D. M. G. Newberry have recently begun to make the connections and have proved a number of revealing theorems that tell us something about the kinds of social values and choice mechanisms that are implied when a given statistical measure of inequality is used normatively. It is this literature on the type of SWF or SCF implied by statistical measures that is reviewed here. The review will emphasize the following three issues: 1. What are the concepts of equity inherent in alternative specifications of SWF's? 2. How does the choice, and ordinal or cardinal uses, of alternative statistical measures of inequality relate to a specific SWF or SCF and its implied concept of equity? 3. Of what practical consequences is the pairing of statistical measures with SWF's and SCF's for policy issues in education equity? The discussion will be organized into four parts. Part I will discuss some definitions of technical terms such as orderings,

symmetry etc, used in the SWF and SCF literature. Part II will outline several different formulations of specific SWF's and SCF's, giving a hypothetical example of their possible application to education equity. Part III will describe the relationship between the specific formulations of SWF's and statistical summary measures, and Part IV will evaluate what we know and how useful that knowledge is for thinking about equity in education.

PART I: A DISCUSSION OF THE MEANING OF TECHNICAL TERMINOLOGY USED IN SWF AND SCF ANALYSIS

In order to discuss the formulation of alternative SWF's and their relationship to empirical measures of inequality, it is convenient to use the terminology prevalent in the literature on social choice. The relevant terminology is listed and explained in the following paragraphs. In this section, when giving examples, arguments (elements) of Social Welfare or Social Choice Functions will be labelled "individual well-being". In the case of education, the concern is more likely to be with arguments such as pupil well-being, district expenditures, taxpayer net income etc. The individual well-being will be assumed to occur in "social states", again somewhat inappropriate to the specific case of education. Because alternative equity conceptions in education have not yet been discussed and because this section is an exposition of some technicalities, it is convenient to adopt the conventional terminology.

1. Rankings (quasi-ordering, ordering, ordinal and cardinal)

Basic to any evaluation of social states is the ability to specify how one state compares to another (for example better, worse,

indifferent or non-comparable, how much better etc.). This rating of social states is called a ranking and there are several ways to conceive of the ranking. One of the least demanding ways is to specify a quasi-ordering which requires that the ranking be transitive and reflexive, but not complete. These requirements can be explained using the following example. Suppose one wants to compare the equality of the distribution of individual well-being in three alternative social states, A, B, and C. The comparison can be specified by stating whether a social state is preferred ($\overline{P}$) to another, at least as good as another ($\overline{R}$), indifferent to another ($\overline{I}$), or not comparable ($\overline{NC}$). Not comparable means that it is unknown how the states compare and no decision can be made. The property of reflexivity means that any social state is at least as good as itself, something that might seem obvious. Although rankings that are not reflexive will not be considered, the condition is included here so as to be technically inclusive. Transivity means that if A $\overline{P}$ B $\overline{P}$ C, then A $\overline{P}$ C; also if A $\overline{P}$ B $\overline{I}$ C, then A $\overline{P}$ C. This is an appealing property of consistency, but one that decisions by so popular a method of ranking as majority vote do not meet. It is not required that all states be capable of being compared to all others; some pairs may be non-comparable, thus making the ranking incomplete. Incompleteness is upsetting when one wants an answer about relative equality in all possible situations. However, it may be a fairly realistic depiction of one's ability to rate different social states since some social states may have such different distributions that one can not know how to compare them.

An ordering is a ranking that is in addition to being reflexive and transitive is complete.

A ranking that is ordinal requires that real numbers be assignable to each social state. The real numbers serve only to position each social state with respect to the others and any positive monotonic transformation does equally well. A positive monotonic transformation is a set of new numbers that leaves each state in the same position as the old numbers. For example the ordinal ranking 1, 2, 3, 4, gives the same information as 18, 99, 100, 9000. An ordinal ranking differs from an ordering because the latter involves no numerical scale at all. There are some ways of thinking about rankings, such as lexicographically, that can be translated to an ordering but not to an ordinal scale.[1]

The most demanding and at the same time the most informative ranking is a cardinal one. With cardinality the value of the numbers do matter, up to a positive linear transformation. That is the number represented by (y) can also be represented by (a + by), b>0. What cardinality means is that differences in two levels of well-being can be meaningfully compared. For example, given a cardinal scale, one could say that the difference between levels of well-being of 10 and 20 is twice that of the difference between 40 and 45. A linear transformation will not change the relationship between differences as the following example shows:

$$(a + b20) - (a + b10) = (20 - 10)b = 10b$$

is twice as large as

$$(a + b45) - (a + b40) = (45 - 40)b = 5b$$

Cardinality is useful because it allows quantitative rather than merely qualitative comparisons between changes in well being.

2. Interpersonal Comparability

Even if the strictest requirement on a ranking were met, that of cardinality, there would still be considerable trouble evaluating inequality if the well-being of one individual could not be compared to another. Cardinality (or any other ordering) says nothing about these interpersonal comparisons. The ability to make interpersonal comparisons is needed in order to evaluate situations such as the following. Suppose A is a social state and x1 and x2 are the well-being of individuals X1 and X2. In order to evaluate the social state it would be nice to know how x1 and x2 compare. Even cardinally does not help if it is not known how X1's cardinal scale compares to X2's. Because with cardinality the zero point and the size of the interval is arbitrary, cardinal differences in well-being between two individuals may make use of a different origin and interval for each and so make the interpersonal comparisons nonsensical. For example, suppose X1's cardinal scale reads 100 while X2's reads 400. Nothing can be said about the relationship between x1 and x2. X2 might be better off then X1 if, for example, zero for both is equivalent in well-being and an interval increase of 1 is also equivalent. Or they might be equal if zero for X1 is equivalent in well being to two hundred for X2 and a unit increase in well-being for X1 is equivalent to 2 units of increase for X2. Or X1 might be better off than X2 if zero for X1 is equivalent to 400 for X2 and a unit increase is the same

for both. Without the ability to compare the cardinal scales, the individual well-beings cannot be compared. And without knowledge of how X1 compares to X2 it will be difficult to make value judgements on alternative distributions. Note that even when interpersonal comparison is possible (that is we know how individuals compare one to the other in terms of well-being), still the largest evaluation problem remains. It is still necessary to decide which distributions of well-being are better; this problem may require not only knowledge of which individual is better off, but by how much as well.

3. Symmetry and Anonymity

Symmetry (used synonomously with anonymity) is a condition specifying that it does not matter, in determining social well-being, which individual is in each position in a distribution. The individuals are perfectly substitutable one for the other (there are no gods who must always be best off).

4. Inequality Aversion

An oft-referred to aspect of the comparison of social well-being between several social states or distributions is the response of the social well-being to equal proportional and equal absolute changes in each element in the distribution. If the level of well-being is invariant with respect to proportional shifts in every element, this is referred to as constant relative inequality aversion. If the level of well-being declines with proportional increases in each element, this is referred to as increasing relative inequality aversion. The same relationships with absolute shifts in each element are referred to as constant absolute inequality aversion and increasing absolute inequality aversion.

5. Pigou-Dalton Condition

This condition is named after two scholars who specified it as a property of measurements of inequality.[2] In the context of SWF's it states that any transfer from a better-off to a worse-off person must increase the level of social well-being as long as the transfer does not reverse the order of the persons. The condition has been further refined to specify the degree of sensitivity of social well-being to transfers from better-off to worse-off at various levels of personal well-being. The refinements have not been given specific names, but they can be classified as follows:

a. Greater increases in social well-being if transfers are made at high levels of individual well-being.

b. Greater increases in social well-being if transfers are made at middle levels of individual well-being.

c. Greater increases in social well-being if transfers are made at low levels of individual well-being.

d. Sensitivity to transfers on the same side of the mean.

e. Sensitivity to transfers on same side of the median.

f. Sensitivity to transfers on different sides of mean.

g. Sensitivity to transfers on different sides of median.

6. Concavity

Concavity is a mathematical property of continuous functions; it describes the curvature of the function. In terms of SWF's it indicates how additions to one individual's well-being, given the well-being of everyone else, are to affect social well-being. If a function is strictly concave, then marginal increases in any

individual's well-being, ceteris paribus on everyone elses, will have a diminishing, but positive, impact on social well-being; less and less importance is attached to increases in individual well-being as the level of individual well-being increases. Actually strict concavity is a stronger condition than is needed. Strict quasi-concavity will do equally well.[3] In either case, the "strictness" is needed to rule out no curvature and thus, situations where a marginal increase in any one individual's well-being has a positive constant or increasing impact on social well-being.

7. Separability, Additivity, and Multiplicative Functions

The contribution of an increase in an individual's well-being to the social well-being may either be dependent on the level of everyone else's well-being or it may be independent of everyone else's well-being. If it is independent--that is if an increase in an individual's well-being increases social well-being the same amount no matter if everyone else is desperate or ecstatic--then the SWF is called separable. An example of a separable function is one that is additive. If for example we let x,y, and z represent the cardinal measurement of well-being of person's X,Y, and Z, then a SWF such as the following one is an additively separable:

$$S = x^{\alpha} + y^{\alpha} + z^{\alpha} \qquad \alpha = \text{any constant}$$

The contribution of x to S does not depend on y or z. On the other hand a multiplicative function is not separable as the following example illustrates:

$$S = x \cdot y \cdot z \ ;$$

x's contribution to S will be zero when eigher y or z is zero and will increase as y or z increases, as long as both y and z are greater than zero.

PART II: DESCRIPTION AND CHARACTERISTICS OF SOME ALTERNATIVE SPECIFICATIONS OF SOCIAL WELFARE FUNCTIONS AND SOCIAL CHOICE FUNCTIONS, WITH HYPOTHETICAL EXAMPLES OF APPLICATIONS TO EDUCATION EQUITY

1. Social Welfare Functions Satisfying Rawls' Principle of Justice

John Rawls has conceptualized a now famous principle of justice based on fairness. Without entering into the extensive debate over the relationship of Rawlsian justice to Rawlsian fairness, we will simply summarize the derived justice principle most useful in social welfare analysis. That principle states that the social welfare analysis. That principle states that the social state that makes the worst-off individual best-off is just. This maximin principle can be extended to provide an ordering of social states. In each social state the worst-off individual is identified. Then the worst-off individuals from each social state are ranked vis-a-vis one another across all states. The state with the "best of the worst-off" individuals is the most preferred state and the state with "the worst of the worst-off" individuals is the least preferred state; the others are lined up between according to the "worse-offness" of the worst-off.

The Rawlsian Social Welfare Function has the following characteristics: it is ordinal, interpersonally comparable and symmetric. It does not meet the Pigou-Dalton principle, because none of the individuals except the worst-off matters to the SWF.

Therefore all kinds of transfers from better-off to less well-off can take place without affecting social welfare, as long as the worst-off are not included in those transfers. The Rawlsian SWF exhibits both decreasing relative and constant absolute inequality aversion. This is because the same proportional or absolute change in everyone's well-being will increase the level of the worst-off person. The Rawlsian function is neither additive, nor multiplicative, again because it does not care what happens to anyone but the worst-off individual.[4] It is also not concave.

An example of an application of the Rawlsian principle of justice to education would be the following. Suppose we are interested in evaluating the justice of different distributions of current operating expenditures per pupil (COE). These different distributions might be the ones observed across the states, in the United States, in a given year. Also, suppose that the level of well-being is to be measured straightforwardly by the level of COE. Then the ranking of states, from most just to least just, would correspond with the descending (from high to low) ordering of the lowest COE in each state. Note that this ordering would ignore almost all the information provided by the state distributions and conversely would use only one COE figure (the lowest) for each state. Since states often have over 500 school districts, over 500 pieces of information would be ignored for each state. On the other hand, maximum weight would be placed on the worst-off pupils, which is of course what the Rawlsian principle advocates.

Also note that it would not be possible to determine how far apart the least and the most just state are, because the Rawlsian principle only provides an ordinal, not a cardinal ranking.

2. Elitist or Nietzschean SWF

Just as the Rawlsian SWF ignores all individuals except the worst-off, The elitist SWF ignores all but the best-off. Although clearly not an eqalitarian motivated SWF, the elitist formulation has not been without advocates. The idea behind the elitist function is that society should strive for the highest possible satisfaction (achievement, happiness, etc.) in its members and that the way to assure this is to reward the individual(s) most capable of producing satisfaction (achievement, happiness). The elitist formulation exhibits all the same technical properties as the Rawlsian.

One might apply the elitist SWF to education by simply reversing the Rawlsian example, but this mechanical interpretation would make little intuitive sense. What is needed to provide any appeal at all to the elitist formulation is a definition of best-off that does not depend solely on the level of COE. Suppose that pupils are ranked from best-off to worst-off by their levels of learning, perhaps as measured by scores on achievement tests. It would be likely in such a case that the highest single level of learning would result from an allocation of all resources to the pupil(s) with the highest learning level. This is likely to be true even if pupils with lower learning levels are able to make greater advances for a given level of resources because the desired final result depends both on where the pupils begin and on how they advance. Pupils with high initial levels have a head start. Therefore, the elitist SWF function would rank distributions by the allocation of COE to pupils by level of learning. The

distribution giving the highest COE to the highest learner would be the most equitable and the distribution giving the lowest COE to the highest learner would be least equitable.

This example skims over the problem of determining levels of learning and in particular the issue of whether the levels should be defined according to their current manifestation or modified to hold constant background. If only current manifestation is used, the 'desirable' distribution of COE is likely to be anti-compensatory.

3. Social Welfare Functions Satisfying Sen's Weak Equity Axiom

Amartya Sen has suggested a weak axiom that puts some mild restrictions on Social Welfare Functions and manages to eliminate such a popular one as the Utilitarian-type function.[5] His weak equity axiom (WEA) is stated in terms of the relationship between two individuals, their levels of well-being, and an argument (element) of well-being such as income. The axiom says that if given the same level of income, individual one has a lower level of well-being than individual two, then individual one should be given more income. This axiom provided only a quasi-ordering, where social states can be classified as worse if they do not meet the axiom's requirements and better if they do. However, two social states that both give more income to individuals of type two, are non-comparable. The WEA is symmetric and requires interpersonal comparisons. It gives little guidance on the final dispersion of income or well-being, because it does not say how much more income the disadvantaged individual must receive. The amount could be very small in one state and extremely large in another, but just so more is received, the two states would meet the axiom's

requirements and could not be compared to one another. Despite its evident weakness, the WEA is still useful because it may exemplify some people's intuitive ideas about equity and it does manage to rule out the utilitarian function.

COE's might again be used, in a slightly different way, to illustrate an application of Sen's WEA to education. The WEA forces an evaluation of well-being gained by different pupils from the same level of COE. Suppose we could compare well-being of different classes of pupils such as physically handicapped, educationally handicapped as measured by reading scores below grade level, geographically handicapped as measured by above average resource costs, economically handicapped as measured by average income of parents, and non-handicapped (all others). In addition suppose we were to order well-being gained from the same COE from low to high in the order listed above. Finally suppose we were to classify each student by <u>one</u> group only, that being the lowest welfare one if there were overlaps. The equitable states would be ones for which the level of COE were allocated to pupil groups from high to low consistently with the order listed above. Inequitable states would be all others.

Note that we could go not further than this two way classification. Some equitable states could conceivable distinguish each pupil group by a $1.00 difference, while others had differences of $100 or over. Yet these states would both be equitable and non-comparable. Likewise for the inequitable states, one state might simply have gotten one group out of order, while another might have large dollar differences between groups ordered exactly backwards

om the WEA specifications. These two states would be inequitable and non-comparable to each other.

4. Utilitarian or Benthamite SWF's

One of the most widely used SWF's in economics has been the utilitarian one. This specification of Social Welfare says that the simple sum of individual's welfare is to be the measure of social welfare. If individual i's welfare is represented by $U_i(x)$ (the utility i receives from social state x), then the utilitarian SWF equals $\sum_{i=1}^{n} U_i(x)$, where n = number of people in the society.[6]

This specification of the SWF is additive, separable and symmetric. It is also a cardinal measure and uses interpersonal comparisons as can be seen by the following description. One way to compare Social Welfare in state x to that in state y is to calculate the difference in utility received by all i in x and y. If the sum of the differences is positive, x is preferred to y. The losses of any individuals must be compensated by the gains of other individuals; losses and gains across individuals are measurable and comparable.

The utilitarian SWF is not necessarily concave; whether it is or not depends upon the specification of the utility functions of the individuals (the $U_i(x)$'s). A very common assumption is that individual utility is a function of income for consumption and is concave with respect to that income (diminishing marginal utility). In this case the SWF will be concave with respect to income also.

Even with the assumption of concave individual utility functions, the SWF does not necessarily meet the Pigou-Dalton condition because some individuals may derive more utility from each given level of income than other individuals. In that case, adding income to any one individual will increase Social Welfare at a diminishing rate, but switching income from a high income individual could result in a larger subtraction of individual utility (and therefore social welfare) than adding that income to a low income individual will increase individual utility (and therefore social welfare). (Note that this example also indicates how Sen's WEA may be violated by utilitarianism. The individual with higher utility at any given income level would be given more income by a utilitarian rule not less, as specified by the Sen's WEA.) The common assumption of identical concave individual utility functions would meet the requirements of the Pigou-Dalton condition. In the case of identical utility functions transfers from extremes of the income distribution (very high to very low) will add more to SW than will transfers at similar levels of income. Of course, if individual utilities are identical, Sen's WEA is not applicable.

It is somewhat difficult to devise an illustration of utilitarianism applied to education because the requirement of cardinality is such a difficult one to meet. If we remain with the COE examples we must, similarly to Sen's WEA, evaluate the well-being that is associated with COE. However, dissimilar to Sen, we must do more than order the well-being; we must also assign a cardinally usable number to it. Perhaps the following example would at least be in the spirit of utilitarianism.

Suppose we continue with the five groups of pupils listed in the previous example as being the only ones to have distinguishably different levels of well-being for equal levels of resources. For expcsitional ease, the groups will be labelled 1, 2, 3, 4, and 5 where:

1 = physically handicapped
2 = educationally handicapped
3 = geographically handicapped
4 = economically handicapped
5 = non-handicapped

Now suppose that 5's well-being for a given resource level is assigned a cardinal value of 1.0 and every other group's is represented in relation to 5's. Then, as an illustration, suppose the other groups are assigned cardinal values of well-being as follows.

Wb1 = 1/5

Wb2 = 1/3

Wb3 = 1/2

Wb4 = 2/3

Wb5 = 1

The Wb's are cardinal representations of levels of well-being at equal resource levels. Now a utilitarian type measure of each state's social welfare can be devised:

$$SW = \sum_{i=1}^{5} (COE_i)\ (Wb_i)\left(\frac{\text{Pupils } i}{\text{Total Pupils}}\right)$$

where Pupils i is the number of pupils in group i. This SWF would be a weighted sum of well-being of 5 groups, weighted by the percent of pupils in each group. The states would be ranked, ordinally and cardinally, highest to lowest in social welfare, by the size of the Social Welfare indicator.

This is, of course, an almost completely perverse example, but it does illustrate one potential of utilitarianism. Because the well-being levels are constant (they do not decline as COE increases) there is no diminishing marginal utility. In addition, the welfare of the worst-off is presumed to be the lowest per dollar of COE. The combination of these two assumptions about utility means that a state may possibly raise its Social Welfare if it spends more COE on group 5 (all others) and less on the worst-off group 1 (handicapped). Whether this is true will partly depend on the percent of pupils in each group, but since group 5 is likely to have the largest percent, the perversity is likely to be true. As with classical utilitarianism, there is nothing in the formulation that necessarily encourages the usual notions of equality.

5. Social Welfare Functions That Avoid Additive Separability

Because the assumptions of additivity and separability are so restrictive and so objectionable to some, there has been some effort devoted to proving theorems, and in general using specifications of SWF's, that avoid those assumptions. A common alternative specification is that the SWF is symmetric and strictly quasi-concave. Implicity this SWF requires interpersonal comparisons. It need not be defined in terms of individual utilities, but rather can be a direct function of income (or some other variable of concern). Its appeal is the avoidance of additive separability.

This formulation will be referred to in part III, on measurement, where its applicability to education will be more obvious.

6. Some Examples of Social Choice Functions

Many people, particularly economists, have been reluctant not only to make the assumptions of additivity and separability, but also those of cardinality and interpersonal comparability. The literature on social choice has avoided all of these assumptions by concentrating on the problems of how to devise rules for combining individual orderings into social orderings or quasi-orderings.

The basic idea in the social choice formulation is to begin with a group of 'n' individuals and 'r' social states and to assume that each of the 'n' individuals has an ordering of the 'r' states. Then those 'n' orderings of 'r' social states are combined into one social ordering of the 'r' states with certain specifications on the characteristics of the rule generating the social ordering. Finally, it is seen if there exist any rules of choice which permit all the specifications to be met.

The most famous example of this type of model is demonstrated by Arrow's impossibility theorem. Kenneth Arrow (1951) states four specifications that the rule generating the social ordering must meet. 1. The rule must work for every possible configuration of individual perferences. For example it cannot be restricted to a configuration of preferences that results in unanimous agreement on all choices. 2. If every individual prefers one social state to another, than so must the social ordering. 3. The social ordering cannot be based on one individual's orderings; it cannot be dictatorial. 4. Social choice on two alternatives must depend on individual choices on those two alternatives only and not on other "irrelevant alternatives".

These four specifications seem mild and reasonable to most people. Arrow proves there is no social rule that can meet all four requirements.

Much of the effort in the social choice field has been devoted to finding combinations of "desirable" or "reasonable" specifications that can result in rules generating social orderings or in finding which conditions can be met by a specific rule. For example, majority vote is a popular rule and although it does not meet Arrow's four specifications, there are other combinations it does meet.[7]

The strengths of the Social Choice approach are first the attention it pays to how the social ordering is generated and second its usual avoidance of strong assumptions on interpersonal comparisons, cardinality, separability etc. Its weaknesses for those who are interested in the specification and measurement of equity are that the results generally identify a process rather than an outcome and that there is no way to translate the results into measures of equity. Of course, one can clearly conceive of measuring equity by processes, not outcomes, as the following example illustrates.

Although a bit strained, it is perhaps possible to devise an education example in the spirit of SCF's (without use of the technical theorems) by concentrating on the process of revenue generation. We might begin by stating specifications we would want the state-local choice of COE to meet. For example, the following might seem appealing in the late 1970's:

1. Local districts must not have a COE imposed (non-dictatorship of a sorts).

2. No child should have less than "adequate" level of service defined in terms of a COE figure.

3. The decision on the tax rate should determine the COE. The Guaranteed Tax Base systems of the 70's for school districts that budget independently of municipalities and have a required minimum tax rate would meet these specifications. Note that this example is only in the spirit of the SCF model. The actual models are much more technical and narrowly defined to decision rules such as majority vote. The usefulness to the analysis of education equity would seem to be the emphasis on process.

A second example using the social choice formulation more strictly could be labelled a Democratic SWF.[8] Under certain conditions the majority vote rule will result in a victory for the voter with the median preferences. If Democratic is used synonomously with majority rule, then a Democratic SWF could be interpreted to mean that social states are ranked according to the level of well-being of the median individual or group. In education this SWF might be applied to the distribution of COE. The distributions would be ranked according to the level of COE spent on the median pupil (or by the median district). Depending on how different financing arrangements stimulate low, middle, and high districts to spend on COE, this SWF could conflict significantly with the Rawlsian (minimax) or Elitist (maximax) specifications. For example, state-local finance arrangements that benefit the middle of the distribution of COE (the median pupil) may turn out to lead to very low spending by the low

spending districts. Similar to the Rawlsian and Elitist SWF, the Democratic one ignores every group but one.

PART III: RELATIONSHIP BETWEEN CHARACTERISTICS OF SWF'S AND SOME SUMMARY MEASURES OF INEQUALITY

A. Some Relatively Non-Controversial Results

Based primarily on the work of Amartya Sen, the following table displays the most general characteristics of the SWF that is implicitly being used when a specific summary measure of inequality is calculated. When the Lorenz curves are non-intersecting, the general characteristics of the SWF are as far as we need go. Although these general characteristics contain value judgments, the judgments are probably ones most people can live with easily. Note that although the SWF is cardinal, the summary measures are only ordinally consistent with it and should therefore be used for ranking only. In the next section, a cardinal measure is discussed.

Table A

Relationship of

Characteristics of SWF[9]	to	Measures of Inequality
1. -strictly quasi-concave -symmetric -cardinal -either same size population and same total income or different size populations and same mean income -Pigou-Dalton		Gini Coefficients from non-intersecting Lorenz curves
2. -strictly concave -symmetric -cardinal -Pigou-Dalton -increasing relative equality aversion -constant absolute equality aversion		Standard Deviation

3.
-not concave at high income levels
-symmetric
-cardinal
-not Pigou-Dalton
-more sensitive to transfers at low income
-constant relative inequality aversion
-decreasing absolute inequality aversion

Standard Deviation of Logarithms

4.
-strictly concave
-symmetric
-cardinal
-Pigou-Dalton
-constant relative inequality aversion
-decreasing absolute inequality aversion
-equally sensitive to transfers at all levels of income

Coefficient of Variation

5.
-not strictly concave
-symmetric
-cardinal
-not Pigou-Dalton
-constant relative inequality aversion
-decreasing absolute inequality aversion

Relative Mean Deviation

3. When Is It Necessary To Be More Precise?

If the Lorenz curves do not cross, there will be no conflict etween the rankings based on the gini coefficient, the coefficient f variation, and the standard deviation of the logarithms. If the orenz curves do cross, there may be conflicts. If the Lorenz rves for two distributions cross once and the gini coefficients e identical, then the distribution with the Lorenz curve that is gher to the left of their intersection will be ranked more equal the standard deviation of the logarithms and more unequal ording to the coefficient of variation.

In most empirical studies, the Lorenz curves will cross and re will be conflicts in rankings. In these cases, if conflicts to be avoided, the characteristics of the SWF must be specified

in more detail than in table A. Anthony Atkinson and others have shown that the SWF when transformed into an index of inequality results in an index exhibiting constant relative inequality aversion must be of the form:[10]

$$SWF(y) = A + B\,\frac{y^{1-e}}{1-e} \qquad \begin{array}{l} e \neq 1 \\ e \geq 0 \\ b > 0 \end{array}$$

$$SWF(y) = A + B \log y \qquad \begin{array}{l} e = 1 \\ B > 0 \end{array}$$

The choice of e is crucial. "As [e] rises we attach more weight to transfers at the lower end of the distribution and less weight at the top. The limiting case at one extreme is [e] $\to \infty$ giving the function $\min_i [y_i]$ which only takes account of transfers to the very lowest income group (and is therefore not strictly concave); at the other extreme we have [e] = 0 giving the linear utility function which ranks distributions solely according to total income"[11]

Once having specified in more detail the SWF, it is possible to construct measures of inequality that will provide an unambiguous ranking and can be used cardinally. Atkinson transforms the SWF into such a measure by means of the concept of the equally distributed equivalent level of income, which he labels Y_{EDE}. This is the per capita income that if equally distributed would give the same level of social welfare as the existing distribution. His measure of inequality is:

$$I = 1 - \frac{Y_{EDE}}{u}$$

or 1 minus of the ratio of the equally distributed equivalent income to the existing mean income. Y_{EDE} is less than or equal to u, so I ranges from 0 [complete equality] to 1 [complete inequality] when e is greater than or equal to zero. Atkinson has a convenient interpretation of I: "If I = 0.3, for example, it allows us to say that if incomes were equally distributed, then we should need only 70% of the present national income to achieve the same level of social welfare (according to the particular social welfare function)."[12]

If we use the SWF, specified previously, Atkinson's I becomes for continuous functions:

$$I = 1 - \left(\int_0^{\max y} \left(\frac{y}{u} \right)^{1-e} f(y)\,dy \right)^{\frac{1}{1-e}}$$

For discrete functions it becomes:

$$I = 1 - \left(\sum_{i=1}^{n} \left(\frac{y_i}{u} \right)^{1-e} f_i \right)^{\frac{1}{1-e}}$$

$e \neq 1$

f_i = proportion of individuals with income equal to y_i.

The advantage of the Atkinson measure is that having fully specified the SWF there are no conflicts in ranking distributions. The disadvantage is that there is no way to specify the value of the "e" in the SWF to suit everyone. As shown in footnote 6, the value of e will result in SWF's ranging from that of Rawl's ($e \to \infty$) to that of Bentham ($e = 0$) to that of the Elitists ($e \to -\infty$).

There is no good way out of the bind. Lorenz curves will undoubtedly cross, making necessary more specific characterizations of the SWF than those in table A.

PART IV: HOW USEFUL IS THIS APPROACH FOR EVALUATING EQUALITY IN EDUCATION?

The approach results in two tangible outputs. First, the examples of SWF and SCF formulations applied to education are some help in thinking about different concepts of equity. Second, Part III's correspondence between the SWF's and statistical measures helps demonstrate that the measures clearly have normative implications. As for the first output, much is left unanswered. There is no discussion in the literature on SWF's and SCF's on how to combine multiple arguments into a choice function, except to say that a single utility index of the several objectives must be formulated.[13] For example, in education the likely concern is not only with COE and their utility to current pupils, but also tax burdens among families, and the utility of COE to non-students and to future citizens. How can these multiple arguments be included in the SWF? The second output on the normative implications of statistical measures is important, but that point can be made more simply and in less space.

Perhaps the most positive output is a third one. The specification of SWF's shows that in order to use a statistical summary measure that provides a complete ordinal ranking, it is necessary to be fairly specific about how to evaluate equity. This specificity is likely to go beyond what many people feel sure about. On the other hand, a weaker specification such as Sen's WEA is easier to agree on, but of course provides only an incomplete ordering. The lesson may be that the use of summary measures, especially when used cardinally, asks too much of our abilities to think about and agree upon conceptions of equity. We may be on firmer conceptual ground with less complete measures.

FOOTNOTES

. See James Quirk and Rubin Saposnik, Introduction to General Equilibrium Theory and Welfare Economics, McGraw-Hill, New York, 1968, pp 17 and 18.

. The two scholars are Hugh Dalton and A. C. Pigou.

. A technical definition of these terms is clearly described in Amartya Sen, On Economic Inequality, W. W. Norton, New York, 1973, pp. 52 and 53.

. See footnote 6.

. Amartya Sen, op. cit., pp. 18-23.

. Sidney Alexander, "Social Evaluation Through Notional Choice", Quarterly Journal of Economics, (QJE), V88, No. 4, November 1974, has formulated the Rawlsian, Elitist, and Utilitarian SWF's as versions of the same SWF with different choices for one of the parameters. A slightly modified version of his presentation is the following:

Let: U_{ij} = utility of $_i$th individual in $_j$th state

n = number of individuals

W_j = welfare indicator in $_j$th state

$$W_j = \left[\sum_{i=1}^{n} U_{ij}^{a} \right]^{\frac{1}{a}}$$

Utilitarian SWF: $a = 1$, $W_j = \sum_{i=1}^{n} U_{ij}$

Rawlsian SWF: $a \to -\infty$, $W_j = \lim_{a \to -\infty} \left[\sum_{i=1}^{n} U_{ij}^{a} \right]^{\frac{1}{a}}$

$$= \min_i (U_{ij}); \text{ (maximin)}$$

Elitist SWF: $a \to \infty$, $W_j = \lim_{a \to \infty} \left[\sum_{i=1}^{n} U_{ij}^{a} \right]^{\frac{1}{a}}$

$$= \max_i (U_{ij}); \text{ maximax.}$$

Footnotes continued

The difference in the SWF's is the choice of the parameter a. When a = 1, only the total welfare, not how it is distributed, matters. When $a \to -\infty$, the total does not count at all, only the distribution is important. When $a \to -\infty$, all the utilities above the smallest will go to zero before the smallest, leaving the smallest to determine W_j. When $a \to \infty$, again only the distribution matters. In this case, the largest utility will dominate as $a \to \infty$ and the largest utility will therefore determine W_j. Rawls does not accept this formulation because he sees the three SWF's as distinctly different conceptually and thinks it is innaccurate to present them as simply different choices of a mathematical paremeter. (John Rawls, "Reply to Alexander and Musgrave", QJE, V88, No. 4, Nov. 1974, pp. 643-646.

7. See Amartya Sen, Collective Choice and Social Welfare, Holden-Day Inc, San Francisco, 1970, Chapter 10.

8. Robert Cooter and Elhanan Helpman, "Optimal Income Taxation for Transfer Payments Under Different Social Welfare Criteria", QJE, Nov. 1974, pp. 656-70.

9. SWF defined as a function of income not utility. If SWF is function of utility, then SWF is ordinal.

10. This SWF is the same one used by Sidney Alexander, as described in footnote 6. The parallel will be shown more clearly in a few paragraphs. Alexander uses a = 1 - e, as the crucial parameter.

11. Anthony B. Atkinson, "On The Measurement of Inequality", Journal of Economic Theory, V 2, 1970, p. 257.

12. Atkinson, op. cit., p. 250.

13. Robert Cooter and Elhanan Helpman, for example, are able to combine the two arguments of tax burdens and working hours into a single utility index by making utility a function of income and leisure. Income in turn is a function of taxes paid and leisure is equal to all non-work hours.

REFERENCES

Alexander, Sidney; "Social Evaluation Through Notional Choice", Quarterly Journal of Economics, V88, No. 4, November 1974.

Arrow, Kenneth; Social Choice and Individual Values, Wiley, New York, 1951.

Arrow, Kenneth; "A Utilitarian Approach to the Concept of Equality in Public Expenditures", Quarterly Journal of Economics, V58, August 1971, pp. 409-15.

Arrow, Kenneth; "Some Ordinalist-Utilitarian Notes on Rawl's Theory of Justice", Journal of Philosophy, VLXX, No. 9, May 1973.

Atkinson, Anthony; "On the Measurement of Inequality", Journal of Economic Theory, V2, 1970.

Cooter, Robert and Helpman, Elhanan; "Optimal Income Taxation for Transfer Payments Under Different Social Welfare Criteria," Quarterly Journal of Economics, November 1974.

Dasgupta, P.; Starrett, David; Sen, Amartya; "Notes on the Measurement of Inequality", Journal of Economic Theory, 1972.

Musgrave, Richard A.; "Maximin, Uncertainty, and the Leisure Trade-off", Quarterly Journal of Economics, V88, No 4, November 1974

Newberry, D.M.G.; "A Theorem on the Measurement of Inequality", Journal of Economic Theory, V2, 1970.

Quirk, James and Saposnik, Rubin; Introduction To General Equilibrium Theory and Welfare Economics, Mc-Graw Hill, New York, 1968.

Rawls, John; A Theory of Justice, Harvard University Press, Cambridge Mass, 1971.

Rawls, John; "Reply to Alexander and Musgrave", Quarterly Journal of Economics, V88, No. 4, November 1974.

Sen, Amartya; Collective Choice and Social Welfare, Holden-Day, San Francisco, 1970.

Sen, Amartya; On Economic Inequality, W. W. Norton, New York, 1973.

Sen, Amartya and Pattanilk, Prasanta; "Necessary and Sufficient Conditions for Rational Choice Under Majority Decision", Journal of Economic Theory, V1, No. 2, Aug. 1969.

Sheshinski, E.; "Relation Between a Social Welfare Function and The Gini Index of Inequality", Journal of Economic Theory, V4, 1972.

APPENDIX 2

Definitions and Formulas for Measures of Equity

UNIVARIATE MEASURES:

1. Range: The difference between the highest and the lowest observation in a distribution.
2. Restricted Range: The difference between two specific points in a distribution, usually defined in percentiles. A common example is the difference between the 5th and the 95th percentile. A second popular restricted range is the interquartile range, the difference between the 25th and 75th percentiles.
3. Federal Range Ratio: The differences between the observations at the 5th and the 95th percentile divided by the observation at the 5th percentile.
4. Percentage of Students: The number of studetns with observations that fall X% above and below the mean divided by the total number of students.
5. Relative mean Deviation: The sum of the absolute values of the difference between each observation and the mean observation divided by the sum of all the observations. Using the pupil as the unit of analysis and expenditures per pupil as the treatment, the formula is:

$$\sum_{i=1}^{N} P_i \mid \mu - X_i \mid \Big/ \sum_{i=1}^{N} P_i \mu$$

where:

P_i = number of pupils in district i

X_i = mean per pupil expenditure in district i

μ = mean per pupil expenditure over all pupils

N = number of districts

6. Permissible Variance: The sum of the observations below the median divided by the sum that would result if each observation below the median were set equal to the median. The formula, using pupils and expenditures is:

$$\frac{\sum_{i=1}^{J} P_i X_i}{M \sum_{i=1}^{J} P_i}$$

where:

P_i = number of pupils in district i

X_i = mean per pupil expenditure in district i

M = median level of per pupil expenditures in the distribution

J = districts below the median level of per pupil expenditures

7. Variance: The average of the squared deviations from the mean. The formula, using the pupil as the unit of analysis, is:

$$\sum_{i=1}^{N} P_i (\mu - X_i)^2 \Big/ \sum_{i=1}^{N} P_i$$

where:

P_i = number of pupils in district i

X_i = mean per pupil expenditure in district i

μ = mean per pupil expenditure over all pupils

N = number of districts

8. Coefficient of Variation: The square root of the variance divided by the mean.

9. Standard Deviation of Logarithms: The square root of the average of the squared deviations of the logarithms of the observations. The formula, where the pupil is the unit of analysis is:

$$\left[\sum_{i=1}^{N} P_i \quad (\overline{\log X} - \log X_i)^2 \Big/ \sum_{i=1}^{N} P_i \right]^{\frac{1}{2}}$$

where:

P_i = number of pupils in district i

X_i = mean per pupil expenditure in district i

$\log X_i$ = logarithm of the mean per pupil expenditure in district i

$\overline{\log X}$ = mean of the logarithm of per pupil expenditures

$$= \frac{\sum_{i=1}^{N} P_i \log X_i}{\sum_{i=1}^{N} P_i}$$

N = number of districts

10. Gini Coefficient: Derived from the Lorenz curve which is constructed as follows. If the observations are ordered in terms of per pupil expenditures from low to high, they can be plotted on a graph using the percentage of the population on the X axis and the percentage of the expenditures accruing to the population on the Y axis. The plot for a distribution where expenditures per pupil are the same for the entire population will be a 45° line, assuming equal units on each scale. Twenty percent of the population will receive twenty percent of the expenditures, thirty percent of the population will receive

thirty percent of the expenditures, etc. If per pupil expenditures are not distributed equally, then the distribution will be represented by a curve below the 45° line. X percent of the population will receive Y percent of the expenditures and at some point X will be less than Y. The Gini coefficient is then defined as the percentage of the area below the 45° line that is between the Lorenz curve and the 45° line.

There are many formulas (transformations of each other) for the Gini coefficient. One such formula is:

$$\frac{1}{2\left(\sum_{i=1}^{N} P_i\right)^2 \mu} \left(\sum_{i=1}^{N} \sum_{j=1}^{N} P_i P_j \left| X_i - X_j \right| \right)$$

where:

P_i = number of pupils in district i

X_i = mean per pupil expenditure in district i

μ = mean per pupil expenditure over all pupils

N = number of districts

11. Atkinson's Index, using a SWF with E > o and not equal to one. The formula is:

$$1 - \left[\sum_{i=1}^{N} P_i \left(\frac{X_i}{\mu}\right)^{1-E} \Big/ \sum_{i=1}^{N} P_i \right]^{\frac{1}{1-E}}$$

where:

P_i = number of pupils in district i

X_i = mean per pupil expenditure in district i

μ = mean per pupil expenditure over all pupils

N = number of districts

E = value judgment parameter to be set equal to a constant > 0 and not equal to 1.

BIVARIATE MEASURES:

1. Simple correlation between dependent (Y) and independent (X) variables:

$$r = \frac{\sum_{i=1}^{n} y_i \, x_i}{\sqrt{\Sigma y_i^2 \;\; \Sigma x_i^2}}$$

where:

$y_i = Y_i - \bar{Y}$

$x_i = X_i - \bar{X}$

n = number of observations

2. Bivariate slope coefficient from regression $Y = b_o + b_1 X$:

$$b_1 = \frac{\sum_{i=1}^{n} y_i \, x_i}{\sum_{i=1}^{n} x_i^2}$$

3. Elasticity from bivariate regression, defined at means:

$$\eta_{x\,y} = b_1 \cdot \frac{\bar{X}}{\bar{Y}}$$

4. Constant elasticity calculated from regression:

$$\ln Y_i = b_o + b_1 \ln X_i$$

$$\eta_{x\,y} = b_1$$

5. Slope from quadratic regression; defined at mean:

$$Y_i = b_o + b_1 X_i + b_2 X_i^2$$

$$\text{Slope} = b_1 + 2b_2 \bar{X}$$

6. Elasticity from quadratic regression, defined at means:

$$\eta_{xy} = (b_1 + 2b_2 \bar{X})\left(\frac{\bar{X}}{\bar{Y}}\right)$$

7. Slope from cubic regression, defined at mean:

$$Y_i = b_o + b_1 X_i + b_2 X_i^2 + b_3 X_i^3$$

$$\text{Slope} = b_1 + 2b_2\bar{X} + 3b_3\bar{X}^2$$

8. Elasticity from cubic regression, defined at mean:

$$\eta_{xy} = (b_1 + 2b_2\bar{X} + 3b_3\bar{X}^2)\left(\frac{\bar{X}}{\bar{Y}}\right)$$

APPENDIX 3

Taxpayer Equity in School Finance Reform: The School Finance and The Public Finance Perspectives

I. Introduction

The formulation of concepts of equity in school finance reform is complicated and is not a value free exercise. As a result, discussions and analyses of equity are difficult and, at times, confusing. The purpose of this article is to sort out and explain certain conceptions of equity, particularly those known as taxpayer equity.

Many examinations of equity in school finance classify equity concerns into at least two broad groupings or types.[1] One type is generally expressed as a concern for children. Most often children's equity is measured using a univariate dispersion measure such as the coefficient of variation or the Gini coefficient or a more complex measure that incorporates differential student needs. Considerable attention has recently been devoted to developing an analytic framework for assessing the equity implications of education finance reform for children.[2] The other equity type is usually expressed as a concern for the taxpayer. Several different formulations of taxpayer equity have been introduced in an ad hoc manner into the school finance literature and these tend to be somewhat different from taxpayer equity formulations found in the public finance literature. The alterna-

1. See, for example, T.L. Johns and D.A. Magers "Measuring the Equity of State School Finance Programs", Journal of Education Finance, 4, Spring, 1978. They also classify "adequacy" as a concern but we consider adequacy to be a separate concern from equity. See also the Office of Education Regulations that have been written to measure equalization in PL 93-380, in the Federal Register, Tuesday, March 22, 1977.

2. See, for example, R. Berne, "Alternative Equity and Equality Measures: Does the Measure Make A Difference" in E.O. Tron, ed., Selected Papers in School Finance, 1977, Washington, D.C.: U.S. Office of Education, forthcoming and L.S. Friedman and M. Wiseman, "Understanding the Equity Consequences of School Finance Reform", Harvard Educational Review, 48, May, 1978.

tive formulations of taxpayer equity have led to some confusion in the school finance literature and in this article we intend to isolate the different equity formulations so that, hopefully, this confusion can be reduced. Before a large amount of empirical work is undertaken to measure taxpayer equity it is important to be clear on its conceptual underpinnings.

There are multiple formulations of taxpayer equity in school finance and some of the confusion in the literature stems from a failure to recognize this. On the one hand, there are cases where taxpayer equity is equated with an ambiguous formulation such as "equal tax rates", a formulation that is equitable only by definition. On the other hand, less ambiguous formulations such as "equal yield for equal effort" are utilized without reference to other, equally plausible equity formulations. A useful distinction that has been introduced into the school finance literature is the distinction between ex ante and ex post taxpayer equity and these formulations can be used to identify conceptually different formulations of taxpayer equity.[3]

Ex ante taxpayer equity is generally evaluated by examining the characteristics of a school finance plan or scheme while ex post equity involves an assessment of the actual spending patterns that result from a school finance plan and the school districts' response to that plan. The distinctions between ex ante and ex post can be made clearly by relating these alternative formulations of equity to the District Power Equalizing (DPE) school finance plan. The DPE, by its nature, produces a situation where, if every district were to levy identical

3. The terms ex ante and ex post have been used explicitly or implicitly by S.M. Barro, "Alternative Post-Serrano Systems and Their Expenditure Implications" in J. Pincus, ed. School Finance in Transition, (Cambridge, Mass.: Ballinger, 1974); M.S. Feldstein, "Wealth Neutrality and Local Choice in Public Education", American Economic Review, 65, 1975; L.S. Friedman, "The Ambiguity of Serrano: Two Concepts of Wealth Neutrality", Hastings Constitutional Law Quarterly, 4, Summer, 1977; L.S. Friedman and M. Wiseman, "Understanding the Equity Consequences of School Finance Reform".

tax rates, equal spending would result. Since this is an assessment of the plan, rather than the results of a plan, equity formulations that are concerned with the presence of DPE or equal yield for equal effort are ex ante formulations. If equity is assessed by examining what districts actually spend under the DPE (or in other words how they set their tax rates) and the spending is related to the districts' ability-to-pay, the formulation would be considered an ex post taxpayer equity one. Thus, some of the confusion in the school finance literature stems from a failure to make the ex ante/ex post distinction.

Formulations of taxpayer equity in school finance could, by their nature, introduce an additional confusion since they are different from the formulations of taxpayer equity commonly found in the public finance literature. For example, school finance formulations of taxpayer equity utilize either spending or effort related to wealth while public finance formulations normally examine either net benefits or costs and benefits in combination with income.

In the sections to follow we will further elaborate the distinctions among different formulations of taxpayer equity. First, we approach taxpayer equity from the education or school finance perspective and we examine the conditions under which a DPE system leads to taxpayer equity, ex ante or ex post. Second, we introduce notions of taxpayer equity that are more consistent with public finance views of taxpayer equity but that can and have been applied to education. In both cases we discuss alternative equity formulations and illustrate these with examples.

II. Taxpayer Equity From the School Finance Perspective

Historically in school finance, concern for taxpayer equity has evolved simultaneously with the DPE so that it is natural for the two to be equated, at

least in some people's minds.[4] Therefore, in this section we examine under what conditions a DPE could be considered equitable from a taxpayer's point of view. But first we need to explain several additional aspects of taxpayer equity.

One way to examine taxpayer equity is to ask what different taxpayers give up in order to obtain education. When we measure what people give up in order to obtain something else we can express what they give up as a price, a ratio of exchange between two goods. In well-established markets, money serves as a medium of exchange and prices are expressed as a ratio of money foregone to one unit of a good received. Once the money price of two goods is established, it is possible to derive the ratio of exchange between two goods directly. For example, if the price of strawberries is $1.00 foregone for one pint of strawberries received, while the price of raspberries is $1.50 foregone for one pint received, then the price of raspberries could be stated as 1.5 pints of strawberries. Since for public education people do not "give up" something as they do when they purchase strawberries but instead give up resources via taxes, we can refer to the price facing a taxpayer as a tax price. In the case of a tax price, the relevant ratio of exchange for any individual taxpayer is the number of tax dollars that must be paid in order to obtain a unit of a public good or service or the ratio of exchange between one public good or service and another.

Before we illustrate the several tax prices that are relevant and relate tax prices to the DPE, the connection between tax prices and equity needs to be shown. For this discussion we will define equity as the equal treatment of equals (horizontal equity) and the unequal treatment of unequals (vertical

4. The book, J.E. Coons, W.H. Clune, and S. Sugarman, "Private Wealth and Public Education, (Cambridge, Mass,: Belknap Press, 1970) exemplifies this evolution.

equity). Thus, for horizontal taxpayer equity we can inquire whether taxpayers who have the same ability-to-pay face the same tax price and for vertical equity we can assess whether taxpayers who have a greater ability-to-pay face appropriately higher tax prices. Note that these formulations of taxpayer equity are ex ante; tax prices do not depend upon individuals' behavior but are determined by the school finance system.

As an alternative to these ex ante formulations the relationship of school spending to taxpayers' ability-to-pay, or ex post equity, could be examined. In this case, horizontal equity would require that taxpayers with the same ability-to-pay receive the same spending and vertical equity could require that those with a greater ability-to-pay do not receive larger amounts of educational spending than those with a lesser ability-to-pay. Note that in this ex post formulation tax prices do not enter into the evaluation of equity. However, tax prices are relevant to ex post equity considerations in an indirect way.

When we assess ex post taxpayer equity the actual spending decisions of the districts are examined, but those spending decisions result from the prevailing school finance plan and the district's response to that plan. The assessment of ex post equity can, of course, be made without reference to the plan. However, if those concerned with ex post equity desire to change the results or spending patterns, they must alter the school finance plan. At the same time, the link between a given plan and desired results is not simple since most plans allow a fair degree of leeway for individual district response. Thus, those concerned with movement toward ex post equity must be able to predict (or model) how districts will respond to school finance plans and this is where tax prices fit in since it can be shown theoretically and empirically that districts

respond to tax prices.[5]

In the remainder of this section we will show the way in which tax prices can be formulated, the assessment of ex ante equity in terms of tax prices, and the effect of a DPE on tax prices. Also, we will indicate how tax prices can affect district responses to a finance plan and how a DPE may predictably lead to inequity, ex post, on account of tax prices and the particular ability-to-pay measure utilized in school finance.

We begin by describing six simplifying assumptions that make the illustration of the way that tax prices are formulated and related to taxpayer equity clearer. These six assumptions are as follows:

1. The local tax price, or the tax dollars paid directly to the local school district per unit of service, are the subject of this analysis.

2. All households face identical resource costs (or private market purchase prices) for education services. The same is true for other public goods and for private goods. This is clearly a simplifying assumption and places where it affects the conclusions will be stated.

3. Each household in a district receives identical levels of education services and identical levels of all other public goods. In order for this assumption to be valid for education either each household would have to contain identica numbers of children or the 'externalities' associated with education would have to compensate those households with fewer children. Any other assumption unnecessaril complicates the analysis of equity and tax prices without changing the general conclusions.

4. Education and other public goods are financed at the local level by means of a locally determined single rate property tax on 100% of market value.

5. The school district and the municipality that provides other local public services are coterminous.

6. The DPE plan guarantees a tax base of $50,000 per family or $50 per family for each mill of property tax levied. Pre-DPE, it can be assumed that the state government allocates state aid by means of a flat grant per pupil.[6]

---

5. Note that districts respond differently according to other characteristics besides tax prices including income and the preference for education. See M.S. Feldstein, "Wealth Neutrality..."; N.W. Grubb and S.M. Michelson, States and Schools (Lexington, Mass.: Lexington Books, 1974); H.F. Ladd, "Local Education Expenditures Fiscal Capacity, and the Composition of the Property Tax Base", National Tax Journal 29, June, 1975; P.E. Vincent and E.K. Adams, "Tax Base Composition and Family Income in Measuring School District Fiscal Capacity", Paper No. 12, Education Finance Cente Education Commission of the States, Denver, Colorado, March 1978.

A hypothetical example is presented in order to illustrate the calculation of the tax prices and their relationship pre and post DPE to the tax rate, individual home values, and ex ante and ex post equity. The example is composed of three all-residential school districts, each of which contains three houses.[7] The assessed property value for each of the houses is displayed in Table 1.

Three tax prices are calculated, all in terms of the tax bill (or tax bill equivalent in the case of private goods) needed to purchase $100 of the good in question. The amount purchased is standardized to $100 for two reasons. First, the $100 figure allows easy manipulation of tax prices. Second, dollars are often used to represent services in the school finance and public finance literature and given assumption number 2 on identical resource costs across households, the dollars could be converted to units of service that would be identical for all households, without changing the conclusions.[8]

The three tax prices for each household in each district before the initiation of the DPE plan are presented in Table 2. In addition, the mill rate that would be required to raise the $100 is also included in parentheses next to each tax price. P_E is the tax bill for each household that is incurred when education spending per family is increased by $100. P_B is the incremental tax price required to finance $100 of other public goods and services per family and P_R is the incremental tax price equivalent for $100 of private goods and services

6. We will refer to education per family rather than child. By assumption 3, they are identical and the family reference clarifies the analysis.

7. The existence of non-residential property further complicates horizontal and vertical equity comparisons. The authors have worked out examples with districts containing non-residential property, but these examples are not included here because they are not essential to the arguments.

8. For example, it is possible to convert either to $1 of services by dividing all results by 100 or to real units by dividing the $100 of services by a resource cost index.

Table 1

Hypothetical Assessed Property Values By District

	District 1	District 2	District 3
House 1	$10,000	$15,000	$ 30,000
House 2	20,000	20,000	40,000
House 3	30,000	40,000	50,000
Total District Assessed Value	$60,000	$75,000	$120,000
Assessed Value/Household or Family	$20,000	$25,000	$ 40,000

Table 2

Local Tax-Prices Per $100 of a Good or Service

Pre-DPE

	District 1	District 2	District 3
	House 1 ($10,000)	House 1 ($15,000)	House 1 ($30,000)
P_E	$50 (5 mills)	$60 (4 mills)	$75 (2.5 mills)
P_B	50 (5 mills)	60 (4 mills)	75 (2.5 mills)
P_R	100 (10 mills)	100 (6.67 mills)	100 (3.34 mills)
P_E/P_R	.5	.6	.75
P_B/P_R	.5	.6	.75
	House 2 ($20,000)	House 2 ($20,000)	House 2 ($40,000)
P_E	100 (5 mills)	80 (4 mills)	100 (2.5 mills)
P_B	100 (5 mills)	80 (4 mills)	100 (2.5 mills)
P_R	100 (5 mills)	100 (5 mills)	100 (2.5 mills)
P_E/P_R	1	.8	1
P_B/P_R	1	.8	1
	House 3 ($30,000)	House 3 ($40,000)	House 3 ($50,000)
P_E	150 (5 mills)	160 (4 mills)	125 (2.5 mills)
P_B	150 (5 mills)	160 (4 mills)	125 (2.5 mills)
P_R	100 (3.34 mills)	100 (2.5 mills)	100 (2 mills)
P_E/P_R	1.5	1.6	1.25
P_B/P_R	1.5	1.6	1.25

per family. P_R does not differ across households and can be used as a convenient numéraire to which the other two prices can be compared. The comparison to the numéraire is effected by forming the ratio of the tax price of education to the private tax price (P_E/P_R) and the ratio of the tax price of other public goods and services to the private tax price (P_B/P_R). Because pre DPE, P_E/P_R and P_B/P_R are equivalent for each household, for equity considerations we need only be concerned with one of them and therefore we will concentrate on P_E/P_R.[9] Throughout the rest of the paper, P_E and P_B will always be compared to the numéraire, P_R. We will henceforth refer to the ratio P_E/P_R as the tax price of education and the ratio P_B/P_R as the tax price of other public goods.

Although conclusions will be discussed after presentation of tax prices under a DPE, we can observe at this point that in terms of a conception of taxpayer equity that relates tax prices to house values, neither horizontal nor vertical equity standards are met pre-DPE. Generally a finance system is judged to be horizontally equitable if equally situated households are treated equivalently. Vertical equity standards are met when households that are unequal are treated in 'appropriately' different ways. Vertical equity usually requires that the ratio of tax burden to some measure of household well-being, such as residential wealth, be at least constant as wealth increases (i.e. a proportional tax schedule). Sometimes a progressive schedule of tax burden is required, where the ratio of tax burden to wealth increases as the level of wealth increases. Pre-DPE, neither the horizontal nor the vertical equity standards are met. Houses of the same value pay different tax prices as seen by looking at P_E/P_R for the two $20,000 houses in Districts 1 and 2 (1 and .8), violating horizontal equity. At the same time, a

9. Note that P_E/P_R and P_B/P_R are identical pre-DPE because education and other public goods are financed identically at the local and state level.

house of lower value pays a higher tax price than one of higher value as can be seen by looking at P_E/P_R for the $30,000 house in District 1 (1.5) and P_E/P_R for the $50,000 house in District 3 (1.25). Later, in section III, we will present a more comprehensive conception of taxpayer equity, but for those who are interested in a conception that relates tax price to household residential wealth, the pre-DPE system is not equitable.[10]

Table 3 presents the local tax prices for each household in each district after the initiation of the DPE system for education with a $50,000 per family (child) guaranteed base. This time P_E/P_R and P_B/P_R are not identical for each household. The significance for ex ante taxpayer equity of the difference in P_E/P_R and P_B/P_R for each household is dependent on a value judgement. The value judgement determines whether education should be considered a uniquely important service, qualitatively different from other publicly provided services. If it is so considered, then ex ante equity evaluators may be content to look at the tax price of education in isolation from the rest of the financial environment facing a household. In such a case, the DPE provides ex ante taxpayer equity according to the limited conception that relates tax prices to house values. Houses of the same value pay exactly the same price for $100 of education (horizontal equity) and higher priced houses always pay a higher tax price for $100 of education such that the ratio of tax price to household residential wealth is constant and a proportional tax schedule results (one version of vertical equity).

The existence of ex ante horizontal and vertical taxpayer equity post-DPE when education is considered a unique good, is consistent with the equity concept that is measured by the "equal yield for equal effort" criterion.[11]

10. Although this conclusion is based on a hypothetical example, its validity in the real world is assured as long as school districts are not perfectly homogeneous with respect to distributions of residential wealth.

11. If vertical equity is defined to mean a proportional tax schedule, as used above, then the existence of ex ante horizontal and vertical taxpayer equity post-DPE, when education is considered a unique good, is precisely identical to "equal yield for equal effort".

Table 3

Local Tax-Prices Per $100 of a Good or Service

Post-DPE

	District 1	District 2	District 3
	House 1 ($10,000)	House 1 ($15,000)	House 1 ($30,000)
P_E	$20 (2 mills)	$30 (2 mills)	$60 (2 mills)
P_B	50 (5 mills)	60 (4 mills)	75 (2.5 mills)
P_R	100 (10 mills)	100 (6.67 mills)	100 (3.34 mills)
P_E/P_R	.2	.3	.6
P_B/P_R	.5	.6	.75
	House 2 ($20,000)	House 2 ($20,000)	House 2 ($40,000)
P_E	40 (2 mills)	40 (2 mills)	80 (2 mills)
P_B	100 (5 mills)	80 (4 mills)	100 (2.5 mills)
P_R	100 (5 mills)	100 (5 mills)	100 (2.5 mills)
P_E/P_R	.4	.4	.8
P_B/P_R	1	.8	1
	House 3 ($30,000)	House 3 ($40,000)	House 3 ($50,000)
P_E	60 (2 mills)	80 (2 mills)	100 (2 mills)
P_B	150 (5 mills)	160 (4 mills)	125 (2.5 mills)
P_R	100 (3.34 mills)	100 (2.5 mills)	100 (2 mills)
P_E/P_R	.6	.8	1
P_B/P_R	1.5	1.6	1.25

The consistency of the two concepts can be seen with the help of Table 4 which shows the wealth per child for each district and the spending per child (equal yield) that would result from a 2 mill tax rate (equal effort). All three districts would spend $100 per child if each levied 2 mills. Table 4 also shows the local contribution and the state contribution to the spending per child. In section III on taxpayer equity from a public finance perspective, it will be shown that the method of financing the state share, and in particular the tax incidence of the particular revenue sources used at the state level, are important. In the ex ante school finance conception stated either as horizontal and vertical equity or equal yield for equal effort, the particular state taxes used to raise the revenue for state aid are not relevant.

If the equity evaluator does not think that education is unique but rather that it is one among many beneficial goods and services and that a taxpayer's entire range of options is important to consider, then both the tax price of education and the tax price of other public goods and services must be considered together. In such a case, ex ante horizontal equity no longer exists because houses of the same value pay the same price for education but different prices for other public goods and services. For example, the $20,000 houses in Districts 1 and 2 in Table 3 both pay $P_E/P_R = .4$ for education, but in District 1 the $20,000 house pays $P_B/P_R = 1$ for other public goods and services and in District 2 the $20,000 house pays $P_B/P_R = .8$ for other public goods and services. The house in District 2 is better off when both prices are considered (no horizontal equity). Likewise higher-valued houses (house 1 in District 3, Table 3, for example as compared to house 2 in Districts 1 and 2) may pay more for education but be more than compensated by a lower price for other public goods and services (no vertical equity).

Table 4

Equal Yield For Equal Effort and
Ex Ante Horizontal and Vertical Taxpayer Equity

	District 1	District 2	District 3
Wealth per child	$20,000	$25,000	$40,000
Tax rate in mills	2	2	2
Spending per child	$100	$100	$100
Local taxes per child	$40	$50	$80
State taxes per child	$60	$50	$20

The equal yield for equal effort criterion will not capture the inequity that may be perceived as a result of differential tax prices of other public goods and services. As emphasized before, equal yield for equal effort looks at the ex ante horizontal and vertical equity of education alone.

The general conclusions that can be drawn from the presentation of tax prices pre- and post-DPE, in Tables 2 and 3 are the following.

1. Taxpayers face different tax prices for education after a DPE, even though the mill rate for an increment of $100 of education is the same for all taxpayers post-DPE (see P_E/P_R across houses post-DPE, Table 3). The mill rate is not conceptually the same as the tax price.

2. The tax price of education varies positively with house value after the DPE, but ex ante vertical and horizontal equity do not necessarily exist because the tax price of other public goods may be relevant as well. When the two prices are looked at post-DPE, horizontal and vertical equity do not exist (see P_E/P_R and P_B/P_R for house 2 in Districts 1 and 2 and house 1 in District 3, Table 3).

3. The identification of the relevant tax price is also important when the conception of taxpayer equity is an ex post one. Ex post taxpayer equity requires that we know the actual expenditures devoted to each child's education and the relationship of those expenditures to a measure of the characteristic of the taxpayer such as wealth or income. The expenditures per child will be related to the tax price of education because school districts will respond to the tax price (as well as other variables such as income, education levels, and age structure of residents of the district) when determining how much to spend per child. Therefore, in order to understand the ex post household equity consequences of a proposed school finance system, we must understand how the school district will respond to various tax prices in its determination of expenditures per child.

There are two requirements prerequisite to an understanding of school district response to various tax prices established in school finance plans. First the theory or model of local government behavior must be specified. A common theory used currently in the economics literature is that a school district's response is determined by preferences of the median voter.[12] This median voter theory is the one adopted for this discussion. The second prerequisite to understanding school district response is a decision on whether to include only the tax price of education, P_E/P_R, or whether to include the tax price of other goods and services as well, P_B/P_R, in the median voter response models. We will outline the expected influence of each decision on school district response models that predict education expenditures per child.

For expositional purposes we make the following assumptions and then use Tables 2 and 3 to derive the expected expenditure per child that results from median voter response models and the ex post equity implications of the expected expenditure decisions.

1. The house of median value represents the median voter (House 2 in Tables 2 and 3).

2. Households with higher valued houses have larger annual incomes (House 2 in District 3 has a higher annual income than House 2 in Districts 1 and 2).

3. The demand for education by the median voter is a function of tax prices, income, and other variables, such as education levels and age of district residents. The other variables are assumed to be the same for all three median voters in our example, so that educational expenditures will be determined by median voter response to tax prices and income alone. It is assumed that higher tax prices of education result in lower spending; higher prices of other public goods and services result in higher education spending because they are substitutes for education; and higher incomes result in higher education spending because education is, in economic terms, a normal good.

---

12. See, for example, R.E. Wagner, The Public Economy, (Chicago: Markham, 1973) pp. 38-39; T.C. Bergstrom and R.P. Goodman, "Private Demand for Public Goods", American Economic Review, 63, June, 1973; W.A. McEachern, "Collective Median Voter Hypothesis", National Tax Journal, 31, June, 1978.

Given these assumptions, the results we would expect are recorded in Table 5 and discussed below. Pre-DPE we would expect District 2 to spend more than District 1 because the price of education is lower to the median voter in District 2, income is the same for the median voters in both districts, and the tax price of the other public good is the same as the tax price of education for each median voter. We would also expect District 3 to spend more per child than District 1 because the tax prices of education and of other goods and services are identical in both districts for each household, while income in District 3 is higher than in District 1. It is unclear whether District 3 would spend more or less than District 2 because while District 3 has a higher tax price for education and for other goods and services than does District 2, it also has a higher income than does District 2.

Thus, ex post horizontal equity would be violated pre-DPE since houses of equal value (House 2 in Districts 1 and 2) spend different amounts per child (more in District 2). Furthermore, one version of ex post vertical equity is violated since the district with the higher average assessed value per child (District 2) spends more per child than a district with lower assessed value per child (District 1). This finding for ex post vertical equity is not unlike the situation prevailing in most states today.

Post-DPE if the median voter's education decisions do not depend on the price of other goods, then Districts 1 and 2 should spend the same because the tax price of education is the same and income is the same. District 3's spending in relationship to Districts 1 and 2 is unclear because while the tax price of education to the median voter in District 3 is higher, so is income.

If the price of other public goods does influence the median voter's decisions on education, then post-DPE District 1 should spend more than District 2 because while the tax prices of education are the same to the median voters in

Table 5

PREDICTED EDUCATION SPENDING PRE AND POST DPE

Pre DPE			Post DPE (Tax price of education and income influence education spending.)			Post DPE (Tax price of education, tax price of other public goods, and income influence education spending.)		
Education Spending In:	Compared To:	Education Spending In:	Education Spending In:	Compared To:	Education Spending In:	Education Spending In:	Compared To:	Education Spending In:
District 1	less than	District 2	District 1	equal to	District 2	District 1	greater than	District 2
District 1	less than	District 3	District 1	?	District 3	District 1	?	District 3
District 2	?	District 3	District 2	?	District 3	District 2	?	District 3

th Districts, the tax price of the other goods and services is higher in District 1 and thus education would be substituted for the other public goods and services in District 1 to a greater extent than in District 2. District 3's spending per child in relationship to Districts 1 and 2 cannot be predicted because while the tax price of education is higher in District 3 than in either Districts 1 or 2 and the tax price of other goods and services is at least as high as in the other two districts, income in District 3 is also highest.

Economic theory would predict that median voters would respond to tax prices of other goods as well as the tax price of education. Adopting this prediction, the conclusion that District 1 will spend more per child than District 2 post-DPE is most likely. This means that the ranking of districts by expenditures per child reverses the position of Districts 1 and 2 pre- and post-DPE. Ex post horizontal equity is again likely to be violated post-DPE because houses of the same value spend different amounts per child. Ex post vertical equity measured by relating district average spending per child to district average assessed property value across districts may be violated post-DPE depending upon how districts respond to the DPE. (In our example, ex post vertical equity would be achieved for Districts 1 and 2 but the situation for District 3 is uncertain.) Despite the existence of the DPE, tax prices for education for differentially property wealthy taxpayers varies as can their income, both of which affect districts' response, so that there may be a positive or negative relationship between spending and property wealth across districts post-DPE.

The ex post taxpayer equity criterion is one that has been extensively addressed by the public finance literature usually in a more comprehensive manner than the relationship of spending per child to residential wealth of the median voter. We turn now to a discussion of taxpayer equity from a public finance perspective.

III. Taxpayer Equity From the Public Finance Perspective

The previous section identified the tax prices of education before and after the initiation of a DPE system and related those tax prices to a possible conception of ex ante taxpayer equity. The conception was that the tax price of education should exhibit vertical and horizontal equity with respect to household residential wealth. That conception is one that seems to be implied in many discussions of ex ante taxpayer equity in school finance, but it differs from the usual conception of ex post taxpayer equity developed in the public finance literature. The usual conception, and its application to the finance of elementary and secondary education, is presented in this section. In addition, data from two hypothetical distributions of school districts are used to demonstrate the potential difference between ex post public finance equity and ex ante and ex post school finance equity.

The public finance conception of ex post taxpayer equity is based on ability-to-pay and it evaluates the relationship between education spending and taxes on the one hand and income on the other hand, all from the perspective of individual taxpayers. There are two implications of this public finance approach that should be noted with respect to the assessment of taxpayer equity. First, total net benefits defined as education spending on behalf of a taxpayer minus education taxes paid by the taxpayer is made an object of concern, as opposed to a less comprehensive focus on tax prices or education spending alone. There have been valid arguments in favor of analyzing costs (taxes) and benefits (spending) separately, each with respect to income, rather than combining the two into a net benefits measure. This is an alternative that might be appropriate for elementary and secondary education when taxpayers include many people without children and the analyst wishes to consider that education spending benefits

only parents.[13] Second, the measure of ability-to-pay is broadened from residential wealth to income. Although the use of income can be criticized because it does not always include changes in net assets and is generally measured on an annual rather than lifetime basis, still income is closer to a comprehensive ability-to-pay measure than is residential wealth.

A major problem in the public finance approach is how to devise a measure that appropriately combines education spending and taxes with income for each household. One measure that resolves the problem rather well is called Atkinson's Index named after Anthony B. Atkinson, a British economist who devised it.[14] Atkinson's Index has been used in the study of income distributions[15] and has recently been introduced to the school finance literature by Robert Inman in an article that evaluates the equity of alternative school finance plans in the New York metropolitan region.[16]

Atkinson's Index is an appealing measure for three reasons. First it allows explicit expression of equity values through choice of the number assigned to a crucial parameter in the index. If the parameter is set at a very high number, the index becomes consistent with a value judgement that weights the bottom of the distribution much more highly than the middle or top. In fact, if the parameter is set to equal infinity, the index can be interpreted as a mathematical representation of John Rawls' equity criterion that is concerned only with the lowest member of a distribution. On the other hand if the parameter is is set very low, close to zero, the index weights all members of the distribution

13. See S.C. Nelson, "The Equity of Public Subsidies for Higher Education", Papers in Education Finance, Paper No. 5, Education Commission of States, Denver, Colorado, February, 1978.

14. A. Atkinson, "On the Measurement of Inequality", Journal of Economic Theory, , 1970.

15. See A. Atkinson, "On the Measurement...", and A. Sen, On Economic Inequality, (New York: W.W. Norton, 1973).

16. R. Inman, "Optimal Fiscal Reform of Metropolitan Schools", American Economic Review, 68, March, 1978.

equally and becomes consistent with Benthamite utilitarianism.[17] A parameter set between zero and infinity weights the bottom part of the distribution progressively more as the parameter's value increases.

The second appealing characteristic of Atkinson's Index is that it has a convenient interpretation that allows one to specify how much total spending on education could be reduced without a consequent reduction in society' well-being, if more equity were introduced into the system. The interpretation provides a measurement of the cost of inequity.

Finally the index is a good measure because it can easily incorporate information on taxes, spending and income into one summary statistic that can be compared across different distributions. Note that the ex ante and ex post school finance taxpayer equity formulations did not take into account the tax burden imposed by the state taxes used to finance state aid; however, the public finance formulation in general and the specific index we describe below are affected by state taxes. For these three reasons, Atkinson's Index warrants a more detailed explanation and this is provided in the following paragraphs.

The index is constructed in two steps. First, spending, taxes, and income are combined into one number for each household (step one) and then the numbers for each household are combined into a summary statistic (step two). Step one involves specifying a level of well-being, or 'utility' level as it is called by economists, for each household or taxpayer and step two combines the utility levels of all households or taxpayers into a measure of societal well-being, or a social welfare function, and then computes an index of the equity of the societal well-being, our end objective.

17. See S. Alexander, "Social Evaluation Through Notional Choice", Quarterly Journal of Economics, 88, November, 1974.

Step one is accomplished by reducing spending, taxes, and income for each household to a level of well-being or a utility level for each household. There are a number of assumptions that are implied in the derivation of such a utility level including the selection of variables in the index and the functional form the index takes. There are an infinite number of possible utility specifications so that the assumptions are critical. For our example, again drawing on Inman, we include the educational expenditures received by each household (ED_i), the gross annual income of each household (I_i), and the total state and local taxes paid by each household (T_i). Note that the variables could be expanded to include other local, state, and federal goods and services and the taxes paid to finance these other public goods and services; however, we will not pursue this here. By excluding these other goods and services we are assuming that income less taxes paid for education plus education spending are the items that determine household well-being.

After determining the variables that enter into a utility formulation we must specify the mathematical operation, also known as the functional form, that is used to combine the variables to a utility index for each household and again, there are an infinite number of possibilities. A common functional form and the one used by Inman is a utility index for each household (U_i) of the form:

$$U_i = (ED_i)^{\alpha} (I_i - T_i)^{1-\alpha},$$

where α is a parameter that varies between zero and one whose meaning will be explained shortly. This utility index, often called a Cobb-Douglas function in economics, has a number of desirable properties. For example, the index is multiplicative so that utility is a function of education and after (education) tax income and the interaction of these quantities. Thus, zero levels of either education or after tax income yield zero utility. Also, the importance of

education versus after tax income can be incorporated into the utility index by specifying the value of α. An α of one implies that all utility is derived from education; an α of zero implies all utility is derived from after tax income. The value of α can be set as an explicit value judgement or by examining the budgets of households. In the illustration at the end of this section we set α equal to .2 which is approximately equal to the share of education spending in many household budgets.[18]

The second step needed to calculate Atkinson's equity index requires that the household utilities be combined into a measure of societal (or community) well-being where the societal well-being is called "social welfare" and the measure is termed a "social welfare function". Then social welfare can be evaluated in terms of its equity. Again it must be emphasized that the formulation of social welfare functions is based on many assumptions and an infinite number of social welfare functions can be formulated. However, as was the case for the utility function we can specify a social welfare function that has certain desirable properties. A social welfare function (SWF) used quite extensively in the economics literature can be specified as follows:

$$SWF = \frac{1}{n} \sum_{i=1}^{n} \left(A + B \frac{(U_i)^{1-E}}{1-E} \right)$$

where n equals the number of households, A and B are constants, and E is a parameter that will be discussed shortly.[19]

18. There are a number of ways that the utility function could be modified to reflect different judgements on how education and income are combined to achieve utility. For example, the α parameter could vary according to a characteristic of the household, such as income, so that education is weighted more or less heavily for low income households. A constant term could be introduced either multiplicatively or additively and this term could also vary by household to represent different relative efficiencies in producing utility from the same levels of income and education. The constant term could also be related to income levels.

19. Note that the SWF can be computed for a state using all households in the state or average values for each school district.

Although we will not go into all the properties of this social welfare function here, certain characteristics are worth mentioning.[20] First, note that social welfare is determined by aggregating household utilities in an additive function. Second, the parameter E, which can vary from zero to infinity, incorporates equity concerns explicitly into the social welfare function and subsequently into the equity index. The larger the value of E, the more concern is shown for the lower end of the distribution of utilities. For example, if E is permitted to be infinity (in the limit) the social welfare function will be consistent with Rawls' maximin principal. If E is set equal to zero, the social welfare function is utilitarian. Thus, the advantage of this social welfare function is that equity concerns can be expressed explicitly by specifying a desirable value or alternative values of E.

Finally, we reach a point where the index of equity can be constructed and we call that index Atkinson's Index (I). Atkinson's Index is derived from the utility and social welfare functions and can be specified as follows:

$$I = 1 - \left[\sum_{i=1}^{n} \left(\frac{U_i}{\bar{U}} \right)^{1-E} \right]^{\frac{1}{1-E}},$$

where U_i, n, and E are described above and $\bar{U}$ is the mean utility levels of all households.[21] Atkinson's Index ranges between zero and one and despite its computational awkwardness it is derived so that it has a reasonable interpretation. The value of Atkinson's Index can be interpreted to mean that if utilities were equally distributed then we would only need 1-I, where 1-I is a fraction of total utility, to achieve the same level of social welfare as we now experience.[22] When I = .25 for example, it means that the same level of social welfare would be obtained if 75% of the total utility is redistributed equally among all households. Thus, when I equals zero it represents complete equity and when I equals one, complete inequity.

It should also be noted that I displays constant relative inequality aversion al-

---

20. See A. Atkinson, "On the Measurement..."

21. The mathematical derivation is available from the authors on request.

22. See A. Atkinson, "On the Measurement..."

though it is not always sensitive to mean preserving transfers.[23] Also, while the assumptions and computations needed to derive I are somewhat complex, the actual calculation of I is rather straightforward.

At the beginning of this section we pointed out several conceptual differences between a public finance formulation of taxpayer equity such as Atkinson's Index and the school finance formulations of ex ante and ex post equity. However, these differences can be seen more clearly with an example.

The example is comprised of two hypothetical "states" where each state has six districts, each district has one household (or taxpayer), and each household has one child in school.[24] Both states have a DPE funding system with a $50,000 guarantee level for assessed value of property per child.

The basic financial and fiscal data for the six districts in both states, States A and B, are displayed in Table 6. Education expenditures per child average $900 in both states; since a DPE is in effect the expenditure per child equals the mill rate times $50,000. Total taxes for education equal state plus local taxes. The state tax in State A is a proportional income tax; in State B the state tax is a progressive income tax. The local tax is equal to the mill rate times the assessed value. The tax price for education shown in Table 6 is the percentage of each education dollar that is provided through the local tax system.

These two hypothetical states dramatically illustrate the potential conflict among the various formulations of taxpayer equity explained in the previous sections. Both states are equitable using different school finance formu-

23. See R. Berne and L. Stiefel, "The Measurement of Equity in School Finance with an Expenditure Disparity Measure", Draft report to U.S. Office of Education, July, 1978.

24. These may seem like overly simplistic assumptions, however the number of districts in the state could easily be increased and the single household assumption could be reinterpreted as an average figure for any number of households. We could introduce different numbers of children per household but this would not affect the conclusions we draw from the example.

Table 6

HYPOTHETICAL HOUSEHOLD MODELS

	A						B					
School District	1	2	3	4	5	6	1	2	3	4	5	6
Number of Households	1	1	1	1	1	1	1	1	1	1	1	1
Number of Children per Household	1	1	1	1	1	1	1	1	1	1	1	1
Education Expenditures per Child	$800	850	900	900	925	1,025	700	800	900	900	1,000	1,100
Gross Income per Household	$12,000	14,000	16,000	18,000	20,000	22,000	13,500	14,000	14,500	15,000	15,500	16,000
Assessed Value Of Property per Household	$24,000	35,000	38,000	40,000	40,000	36,439	23,000	28,625	30,444	40,000	44,000	46,000
Local Mill Rate	.016	.017	.018	.018	.0185	.0205	.014	.016	.018	.018	.020	.022
Local Taxes per Household	$384	595	684	720	740	747	322	458	548	720	880	1,012
State Taxes per Household	$180	210	240	270	300	330	135	175	217.5	262.5	310	360
Total Education Tax Bill Per Household	$564	805	924	990	1,040	1,077	457	633	765.5	982.5	1,190	1,372
Tax Price For Education (P_E/P_R)	.480	.700	.760	.800	.800	.729	.460	.573	.609	.800	.880	.920

lations of ex ante taxpayer equity. One version of horizontal tax price taxpayer equity is present in both states since districts with identical valued houses face identical tax prices. Note, for example, that Districts 4 and 5 in State A have the same house value and face the same tax price. A version of vertical tax price taxpayer equity that only considers the tax price of education is exhibited in both states since districts with houses of higher value face higher tax prices. In this case, due to the structure of the DPE, the tax price is proportional to house value. Finally, the ex ante school finance taxpayer equity formulation specified as "equal yield for equal effort" is present in both states since the mill rate determines spending due to the DPE. Thus, according to a number of ex ante school finance taxpayer equity formulations both states can be considered equitable.[25]

Ex post school finance taxpayer equity is normally concerned with the actual relationship between spending and wealth expressed in assessed value of property. There are a number of statistical measures that capture this relationship including the correlation, slope and elasticity where in each case a value of zero represents no observed relationship between spending and wealth. While the elasticity, defined in this case as the slope from the regression, ED = f(Wealth), multiplied by mean wealth divided by mean spending, may be a preferable measure, the elasticity, slope, and correlation all calculated from the simple regression, ED = f(Wealth), are displayed for States A and B in Table 7.[26] For all three school finance ex post taxpayer equity measures State A is judged as more equitable than

25. As pointed out in section II, a DPE for education does not introduce tax price equity for other public goods.

26. For a more in-depth discussion and empirical analysis of various ex post school finance measures see R. Berne and L. Stiefel, "A Methodological Assessment of Education Equality and Wealth Neutrality Measures", Public Policy Research Institute, Graduate School of Public Administration, New York University, and Papers in Education Finance, Paper No. 17, Education Commission of the States, Denver, Colorado, July, 1978.

Table 7

Measures of Ex Post Taxpayer Equity For States A and B

	State A	State B
1. School Finance Measures		
a. Correlation ED=f(Wealth)	.6174	.9341
b. Slope ED=f(Wealth)	.0078	.0142
c. Elasticity ED=f(Wealth)	.3083	.5577
2. Public Finance Measures		
a. Atkinson's Index E=1.1	.0175	.0020
b. Atkinson's Index E=8.0	.1133	.0145
c. Atkinson's Index E=400	.2527	.0887

State B although, compared to complete equity, both states are inequitable. This is clearly at odds with the findings from ex ante school finance taxpayer equity for States A and B.

Atkinson's Index of equity can be used to assess the situation in States A and B from an ex post public finance perspective. Recall that Atkinson's Index incorporates information on each district's educational spending, gross income, and local and state taxes for education. Earlier it was pointed out that Atkinson's Index could incorporate different preferences for equity by varying the parameter E and three values of Atkinson's Index with E specified at 1.1, 8.0, and 400 (all with α at .2) are displayed in Table 7. Since equity by Atkinson's Index is defined as zero neither State A nor State B is completely equitable from this formulation. Now however, State B is judged to be more equitable than State A for a wide range of values of E, a reversal of the findings from the ex post school finance taxpayer equity formulations.

This example has shown that under rather plausible conditions the alternative formulations of taxpayer equity can yield contradictory results. This reinforces the suggestion that alternative equity formulations should be considered by school finance researchers and policy analysts.

IV. Conclusions

In the 1970's the quest for equity in school finance has been significantly influenced by the courts. Judges and judicial advocates have naturally been drawn toward conceptions of equity that conform with legal standards and sometimes the resultant conceptions have been simplified and somewhat narrow. The simplification is especially evident with respect to conceptions of taxpayer equity because of the existence of a vast public finance literature that has discussed and debated ways to think about and measure taxpayer equity long before the courts were brought

into the school finance cases. As state legislatures continue to respond to the courts and to their own constituents in efforts to reform school finance to achieve greater equity, it is important to step back and think about what we mean by taxpayer equity, what we wish to achieve by reform, and how we will measure the results of reform efforts. This paper has addressed these issued for taxpayer equity.

There are a number of ways to think about taxpayer equity. We have discussed the differences between ex ante and ex post conceptions as well as the differences between the school finance and the public finance approaches. The school finance approach, whether ex ante or ex post, always uses wealth as a measure of ability-to-pay. The ex ante school finance approach, whether seen as the relationship between tax prices for education and house values or equal yield for equal effort includes a measure of both spending and taxes, while the ex post school finance approach relates spending alone to wealth.

The public finance approach is an ex post one that in addition to including a measure of both spending benefits and tax incidence, uses income as the ability-to-pay measure. The public finance approach has much to recommend it, including a broad measure of ability-to-pay, a comprehensive measure of net benefits, and the allowance of local choice. In general, the public finance approach is more comprehensive and relates more easily to the ways in which public sector equity is usually thought about and measured.

An obvious first reaction to the use of the public finance approach is to express concern about the ability to collect the needed data, especially the income data and to some extent the tax data. There are several responses to this reaction. Robert Inman has recently used the approach in his study of the New York metropolitan region and by making use of data on average households has been able to execute the approach.[27] In addition, many of the analysts who have looked at

27. R. Inman, "Optimal Fiscal Reform..."

equity in higher education where the courts have not been so influencial, have used the public finance approach and have been able to accumulate data on income, taxes and benefits.[28] Third, a number of states use income in their measures of district ability-to-pay and in those states the income on a household basis could be obtained, if necessary by a sample.[29] Fourth, the census data are available decennially and may in the future become available quinquennially. Finally, if the public finance approach were a desirable one, then it would be possible to collect the necessary data. Although analysts always need to be creative in combining data availability with conceptual demands, over a longer run period it is also the role of analysts to influence the kind of data that are collected so that desirable measures can be provided.

In the end the choice of an appropriate conception of taxpayer equity is a value judgement. Value judgements need not be entirely individualistic because sometimes there are preferences that are widely held, but a prerequisite to sorting out the value judgements is an understanding of the possible alternative conceptions. This paper has tried to broaden the possible conceptions of taxpayer equity by sorting out the differences in the school finance approach and by introducing the public finance approach.

28. See, for example, W.L. Hansen and B.A. Weisbrod, "The Distribution of Costs and Direct Benefits of Public Higher Education", Journal of Human Resources, Spring, 1969, and the debate in the Journal of Human Resources in subsequent issues.

29. As reported in A. Odden, "School Finance Reform in the States: 1978", Report No. F78-1, Education Finance Center, Education Commission of the States, June, 1978, p. viii, Missouri, Kansas, Maryland, Connecticut, and Rhode Island use a measure of income in the school finance plan.